Chapter 28: Monopoly and Monopsony

28.1: Introduction

The previous chapter showed that if the government imposes a tax on some good that there is a loss of surplus. We show a similar result in this chapter where we consider markets in which on one side of the market there is a single agent – and moreover an agent who can choose the price in the market. Up to now we have been considering markets in which agents have to take the price as given – we have called these markets price-taking or competitive markets, using the terms synonymously. There is clearly a bit of a hidden assumption here – namely that the number of agents in the market is ‘too large’ for any one agent to choose the price. We have given some justification of this in Chapter 2, where we argued that in the competitive equilibrium there was no agent who could ‘break’ the equilibrium price. In this chapter we consider a situation where this hidden assumption cannot possibly be true – namely a market in which there is one agent on one side of the market; clearly he or she can set the price, or, at a minimum, refuse to trade. Obviously, if the agent can set the price, then generally he or she will choose to do so – since doing so will cause an increase in their profits or surplus (as we saw in chapter 8). There are two possibilities – when the single agent is the single seller and when he or she is the single buyer. The first of these is called monopoly, the second monopsony. We consider the two in turn.

28.2: Profit Maximisation for a Monopoly

We repeat the analysis of chapter 13 – with one difference. The firm can choose the price of the good as well as the quantity produced. Obviously the firm can not choose these independently of each other – there is the constraint of the demand to consider. The higher the price that the firm selects the lower is the quantity it can sell; and the higher the quantity it wants to sell, the lower the price that it can charge. Otherwise it faces the same kind of problem as the price-taking firm – it must choose the output (and price) to maximise its profits.

The crucial point for a monopolist is that when the firm increases its output the lower is the price that it can charge. The constraint is the demand curve for the product, which we write in inverse form as

\[ p = f(y) \]

where \( y \) is the output produced and sold\(^1\), and \( p \) is the price that the firm charges for the product. We expect that the relationship is an inverse one – as the price rises the quantity demanded and hence sold decreases.

Let us now do what we did on chapter 13 – that is, construct a graph with the output along the horizontal axis and with total revenue, total cost and total profits on the vertical axis. The cost function is just as it was in chapter 13 – its shape depending upon the returns to scale that the firm has – and we shall discuss it shortly. In the meantime let us consider the shape of the total revenue function.

The revenue of the firm is simply the price multiplied by the output – \( py \). For a competitive firm, for which the price is given and fixed, this, as a function of \( y \) is linear with slope equal to \( p \). However

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\(^1\) There would be no point producing an output if it were not sold.
for a monopolist the price is not given and fixed, but depends upon the output that the firm produces and sells.

If we combine the above two expressions, we find that the revenue of the firm is given by
\[
\text{revenue} = py = f(y) y
\]
(28.1)
Consider this as a function of \( y \). There are two components \( y \) and \( f(y) \) – the first of these increases with \( y \) and the second decreases with \( y \). The net effect depends upon the particular form of the demand function \( f(.) \). Let us consider an important special case.

Consider a linear demand curve:
\[
p = \alpha - \beta y
\]
(28.2)
In a graph with \( y \) on the horizontal axis and \( p \) on the vertical, this has an intercept on the vertical axis of \( \alpha \), a slope of \( -\beta \), and an intercept on the horizontal axis of \( \alpha/\beta \).

Now consider the revenue of the firm. This is \( py \), which gives
\[
\text{revenue} = py = \alpha y - \beta y^2
\]
(28.3)
Plotted in a graph with \( y \) on the horizontal axis and the (total) revenue on the vertical axis, this is quadratic, starting at 0 and then rising at a decreasing rate until \( y = \alpha/2\beta \), and then falling until \( y = \alpha/\beta \), at which point the revenue is zero (because the price is zero), after which the revenue is negative. This is shown in figure 28.1.

For future reference it will be found useful to find the slope of the revenue function – this is the rate at which revenue rises when the output rises. It is given the name of the marginal revenue function. From (28.3) we get
\[
\text{marginal revenue} = d(\text{revenue})/dy = \alpha - 2\beta y
\]
(28.4)
So the marginal revenue function is a straight line when plotted against \( y \), with an intercept on the vertical axis of \( \alpha \), a slope of \( -2\beta \), and an intercept on the horizontal axis of \( \alpha/2\beta \). Note its relationship with the demand curve, which we assumed to be linear: it has the same intercept on the vertical axis and the intercept on the horizontal axis is half that of the demand curve. Note that this intercept is where the marginal revenue is zero and it coincides with the output at which the revenue curve reaches its maximum point.

Let us now return to our main theme. Let us draw a graph of total revenue, total cost and total profit as a function of output. We have already argued that with a linear demand schedule that the total revenue function is quadratic. Let us assume first that the firm has decreasing returns to scale so that its cost function is convex. We thus have figure 28.3. In this the cost function is convex, the revenue function is quadratic (and concave) and the profit function is also concave. It will be seen from the figure that profits are zero when the output is zero, and again when the output is just over 70. In between the profits are positive. Above an output of 70 profits are negative.

The figure shows also the profit-maximising output. This is where the profit function reaches its maximum – or alternatively where the vertical distance between the revenue function and the cost function is maximised. It will be seen that this is where the slope of the revenue function is equal to
the slope of the cost function. (We have drawn in the tangents to the two curves at this profit-maximising point to make it clear.)

Now we can interpret this condition. At the profit-maximising point, the slope of the revenue function must equal the slope of the cost function. Now we know what these two slopes are: the slope of the revenue function is the marginal revenue function; the slope of the cost function is the marginal cost function. So the profit-maximising condition is that

\[ \text{marginal revenue} = \text{marginal cost} \]

A formal and general proof is provided in the Mathematical Appendix to this chapter. In the particular case that we are considering here, we can see from the graph that the profit-maximising output is around 34.5.

As ever we can translate everything from total curves to marginal curves. We get figure 28.4. In this the straight line is the marginal revenue curve: as we have already shown it is a straight line with intercept on the vertical axis 100, slope –2 and intercept on the horizontal axis of 50 (half that of the demand curve for which we took the particular values \(\alpha = 100\) and \(\beta = 1\). The upward sloping curve is the marginal cost curve – we know that this is everywhere increasing if we have decreasing returns to scale. Where the two curves intersect is the profit-maximising point – at an output of around 34.5 - exactly as before.

So we have the condition for the optimum output. The optimal price is then given by the demand function.
28.3: Loss Minimisation

We should be a little careful in applying the profit-maximising condition – it does not guarantee a positive profit, but merely that profit is maximised. This is very easy to see if we take the example above and add to the firm’s costs some new fixed cost – perhaps a government tax – which is sufficiently large to make the profits everywhere negative. What happens? To the total revenue and hence marginal revenue curves – nothing. To the total cost curve – the new fixed costs shifts the entire cost curve up by a constant amount. To the marginal cost curve – nothing – because shifting a curve vertically upwards by a constant amount does not change the slope at any point. So the addition of the fixed costs, while making profits negative everywhere, does not change the figure 28.5 – it looks exactly the same. Is the identified point still a profit-maximising point? Well, yes in the sense that it minimises the losses – which now exist everywhere because of the fixed costs. But there are losses at that point. It may pay the firm to give up production altogether.

28.4: Increasing Returns to Scale

You may recall that for a competitive firm, the optimal output for a firm with increasing returns to scale is infinite. Let us see whether that is still the case for a monopoly. We repeat the analysis above, the only difference being that the cost curve is concave and hence that the marginal cost curve is everywhere decreasing. We start with total costs and revenues on the vertical axis.
We then derive the corresponding marginal curves.
From this we see that there are two points at which marginal revenue and marginal costs are equal – one at around an output of 4 and one around an output of 31. If we look back at figure 28.13 above we see that the first of these is a (local) *loss maximising point* while the second is a *profit-maximising point*. If we examine figure 28.12 carefully, it can be seen that for the first of these the marginal cost curve cuts the marginal revenue curve from above, while in the second the marginal cost cuts the marginal revenue from below. It is clear that we ought to supplement our condition for the profit-maximising point in the following way\(^2\):

\[
\text{marginal revenue} = \text{marginal cost at a point where the marginal cost curve cuts the marginal revenue curve from below}
\]

### 28.5: The ‘Supply Curve’ for a Monopolist

It should be clear that a supply curve for a monopolist does not exist - simply because the monopolist does not take the price as given - but rather takes the demand curve as given.

### 28.6: Producer Surplus

What is the producer surplus under monopoly? Consider figure 28.15. In this figure we have inserted the demand curve (the downward sloping straight line going from (100, 0) to (0, 100), the marginal revenue curve (the downward sloping straight line going from (50, 0) to (0, 100), and the marginal cost curve.

\(^2\) This is the second-order condition – which is proved formally in the Mathematical Appendix to this chapter.
The optimal output is around 34.5 – where the marginal revenue and marginal cost curves intersect. The optimal price is then given by the demand curve and is illustrated in the figure above – it is a price of around 65.5. (Recall that the demand curve is given by $p = 100 - y$.) The total revenue of the firm is the product of the quantity produced, 34.5, and the price, 64.5. It is the area of the rectangle bounded by the optimal price and the optimal quantity. The total costs of the firm to produce the output 34.5 is the area under the marginal cost curve up to the optimal output. It follows therefore that the profit of surplus of the firm is the area between the optimal price, the optimal quantity and the marginal cost curve. The buyer surplus is the area between the optimal price and the demand curve.

It is instructive to compare this situation with that under competition. If the firm was a price-taker and if an equilibrium price was charged, the output would be just over 61 and the price would be a little under 39 – as in figure 28.16.
The buyer surplus would be the area between this price and the demand curve – obviously bigger than the surplus that the buyers get under monopoly. The producer surplus would be the area between the competitive price and the marginal cost curve – obviously smaller than the surplus the producer gets under monopoly. So there is a re-distribution of the surplus – under monopoly the producer gets more surplus and the buyers less. But there is also a reduction in the total surplus – the triangle between the monopoly output, the demand curve and the marginal cost curve is missing under monopoly. Monopoly is inefficient – it leads to a reduction in the total surplus - because there is a reduction in the quantity traded.

28.7: Profit Maximisation for a Monopsonist

We build on the work that we did in chapter 26 – the labour market. We considered there a firm demanding labour in the labour market, though we assumed there that the labour market was competitive and that the firm took the price of labour (the wage rate) as given. In this chapter we consider the situation in which the firm is the only buyer of labour – that is, we assume that the firm is a monopsonist in the labour market.

Recall the decision problem of the firm – to make things simple we assume that the firm is in the short run and it must decide the quantity of labour to employ. In chapter 26 we assumed that the wage rate (the price of labour) $w$ was fixed. If the firm is the only buyer in the labour market then it

\footnote{You should make sure that you understand why I have written ‘obviously’ here.}
must take account of the supply curve of labour. If this is upward-sloping then the more labour it employs the higher the wage rate that it must pay. Let us take a simple case and assume that the supply of labour curve is linear. Specifically:

$$w = \gamma + \delta l$$  \hspace{1cm} (28.5)

where \(w\) is the wage rate and \(l\) the quantity of labour it employs. Now recall that the total cost to the firm of employing \(q\) units of labour at a wage rate \(w\) is given by:

$$\text{total cost} = wl + rK$$

where \(r\) is the price of the fixed input (we are in the short run) and \(K\) is the fixed quantity of the fixed input (capital if you like). For a competitive firm the value of \(w\) is fixed but for a monopsonist it depends upon the quantity of labour purchased. As \(l\) rises so does \(w\) and hence the total cost rises more than proportionately. If we substitute (28.5) into the cost function we can see how the total cost depends upon the employment of labour. We have

$$\text{total cost} = (\gamma + \delta l)l + rK$$

You will see that the total cost is a quadratic (and convex since \(\delta\) is positive) function of \(l\).

For future reference it will be useful to find the marginal cost as a function of \(l\). This is the slope of the total cost function when graphed against \(l\). It is given by the derivative of the total cost with respect to \(l\). We have

$$\text{marginal cost} = \gamma + 2 \delta l$$  \hspace{1cm} (28.6)

Notice that it has the same intercept at the supply of labour curve and twice the slope. (Note the parallels with the fact that the marginal revenue curve for a monopolist has the same intercept as the demand curve and twice the slope.)

Now we are in a position to draw a graph of the total revenue, total cost and total profit of the firm as a function of the amount of labour that it employs. Using the same arguments that we used in chapter 26, the revenue function of the firm – as a function of the amount of labour that it employs must be concave – with a slope equal to the value of the marginal product of labour. (That is, equal to the price of the output of the firm multiplied by the marginal product of labour.) We thus get figure 28.18. In this figure the upper concave function is the total revenue of the firm, the convex (quadratic) function is the total cost of the firm and the remaining curve is the difference between the two – namely the profit function of the firm. It will be seen that the profits reach a maximum where the gap between revenue and cost is maximised – which occurs where the slope of the revenue function is equal to the slope of the cost function.
But we know these two slopes: the slope of the revenue function is equal to the value of the marginal product of labour; the slope of the cost function is what we have called above the marginal cost function. So the profit maximising condition\(^4\) is that

\[\text{the value of the marginal product of labour} = \text{the marginal cost of labour}\]

We see that the optimal quantity of labour to employ is around 28.

We can, once again, translate all the above (which is in totals) to curves involving marginals – which are just the slopes of the total curves. We note that the slope of the total revenue curve is everywhere falling and that the slope of the total cost curve is everywhere rising. In the particular case that we are considering we know that in fact the marginal cost curve is linear - see equation (28.6). We get figure 28.20 from which it is clear that the marginal cost equals the marginal revenue (the value of the marginal product of labour) when around 28 units of labour are employed – exactly as we found above.

\(^4\) A formal and general proof can be found in the Mathematical Appendix to this chapter.
In this figure, the downward sloping line is the marginal revenue function, the lower upward-sloping line the supply of labour function and the upper upward-sloping line (with twice the slope) the marginal cost function. The wage rate that the firm pays is given by the supply of labour of curve at the optimal quantity of labour – in this example a wage rate of just under 0.44.

We can now work out the various surpluses under monopsony. The workers get the area between the wage rate and the supply of labour curve. The firm gets the area bounded by the wage rate, the optimal employment and the marginal revenue curve. What would happen under competition? The wage rate would be such that it equalled the value of the marginal product of labour – so optimal employment would be just under 44 and the wage rate would be around 0.65. Notice that monopsony reduces the employment and the wage rate. The latter of these seems obvious – a sole employer uses pays less because it has the power to do so. The first seems less obvious – but is the inevitable consequence of paying less: given the upward-sloping supply of labour curve, less labour is supplied at a lower wage.

Monopsony also changes the surpluses. Under competition the surplus going to labour is the area between the competitive wage and the supply of labour curve. The surplus going to the firm is the area between the competitive wage and the marginal revenue curve (which is the demand for labour curve under competition). So monopsony lowers the surplus going to labour and increases the surplus going to the firm. But it also has the effect of reducing the total surplus – the triangular area between the monopsony price, the demand curve for labour and the supply curve of labour is now lost – as a consequence of the lower quantity of labour employed. We get the familiar conclusion – *monopsony is inefficient*. It lowers the total quantity of surplus extracted from the market – as a consequence of a lower quantity traded.

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5 The area underneath the marginal revenue curve is the total revenue and the rectangular area bounded by the optimal wage rate and the optimal employment is the (short run) costs – so the difference is the profit – forgetting about the fixed cost.

6 Make sure you understand why this *must* be the case.
28.8: Minimum Wage Legislation

In this situation there is something that the government can do: it can introduce a legally-binding minimum wage. If it does then the obvious thing to do is to set it exactly at the competitive wage level. In this way it forces the monopsonist to act as a competitive firm – and the surpluses are restored to what they would have been under competition. The amount of labour employed is also increased – though, of course, the monopsonist gets a lower profit under such legislation than if it was allowed to pay what it wanted.

28.9: Summary

In this chapter we have considered two situations in which a single agent has the power to set the price: monopoly and monopsony. The first of these is when there is a single seller in a market; the second when there is single buyer.

For a monopolist profit maximisation implies that the marginal revenue should equal the marginal cost at an output where the marginal cost curve intersects the marginal revenue curve from below.

There is no such thing as a supply curve for a monopolist.

Monopoly causes a loss of surplus.

For a monopsonist profit maximisation implies the marginal labour revenue should equal the marginal cost.

There is no such thing as a demand curve for a monopsonist.

Monopsony causes a loss of surplus.

Minimum Wage legislation helps restore the lost surplus.

And you might like to ask whether maximum price legislation for a monopolist helps to replace the lost surplus.

28.10: Monopoly and Monopsony in the same market

You may have been wondering what happens in a market where there is a single buyer and a single seller. We know what happens if we have a single seller and lots of buyers (the seller acts as a monopolist, takes a bigger surplus and so doing reduces the total surplus generated in the market). We also know what happens if we have a single buyer and lots of sellers (the buyer acts as a monopsonist, takes a bigger surplus and so doing reduces the total surplus generated in the market). You may be thinking- “if there is a single seller and a single seller perhaps the two effects cancel out and we get back to the efficient case of perfect competition?".
This seems a sensible conclusion but is difficult to justify. The problem is that we have not yet defined the rules of the trading game in such a market. In a market in which there is a single seller and lots of buyers the obvious rule is easy to state: the seller posts a price and the buyers buy what they want at that price; the seller sells a quantity equal to the aggregate demand at that price. In a market in which there is a single buyer and lots of sellers the obvious rule is once again easy to state: the buyer posts a price and the sellers sell what they want at that price; the buyer buys a quantity equal to the aggregate supply at that price. However, in a market with a single buyer and a single seller, there is no such obvious rule. One possible story is that the buyer and the seller indulge in some kind of bargaining process – they bargain over the price at which they will trade and the quantity which they want to exchange. Economists do have stories as to how bargaining is carried out, though a full discussion of such stories takes us way beyond the scope of this book. We can, however, describe one such story – known as the cooperative bargaining solution. The term ‘cooperative’ may seem a bit odd in a bargaining situation, which is one of conflict, but we shall describe what is meant by this.

In essence it means that the two bargainers (the monopolist and the monopsonist in the example) agree to the rules governing the outcome to the bargaining problem and then ask someone to apply those rules. The rules must appear reasonable to both bargainers.

What might these rules be? One set was proposed by the brilliant mathematician and economist John Nash – the same man who proposed the Nash equilibrium concept in game theory (which we will discuss in Chapter 30), who won the Nobel Prize in Economics and about whom a book and a film, both called A Beautiful Mind, were made. These rules assume that the two bargainers obey the axioms of Expected Utility theory (see Chapter 24) so that they both have a cardinal utility function (invariant up to a positive linear transformation) over the final outcome. Moreover, there is assumed to be some disagreement outcome, which describes the utilities of the two bargainers if they fail to reach an agreement. In the context of the monopolist and the monopsonist, the disagreement point must simply be the situation in which they do not trade and therefore receive zero surpluses. The rules are the following:

1. (Efficiency) The outcome must be efficient in the sense that there is no other outcome which is better for one of the two bargainers and no worse for the other;
2. (Linear Invariance) If the utility function of one or other of the bargainers is linearly transformed into another function then the outcome should be unchanged.
3. (Symmetry) If the bargaining problem is symmetric (that is, the disagreement outcome is the same for both), then so should be the outcome;
4. (Independence of Irrelevant Alternatives) If we have two bargaining problems (Problem 1 and Problem 2) with the same disagreement outcome, and for which the set of possible alternative outcomes in Problem 1 is contained within the set of possible outcomes in Problem 2, and if the chosen outcome in Problem 2 is within the set of possible outcomes of Problem 1, then the chosen outcome in Problem 1 should be the same as that in Problem 2.

Assumption (1) seems innocent and so does assumption (2) – after all the utility functions are unique only up to a linear transformation (changing the units of measurement of utility should not change the outcome). Assumption (3) is more subtle – as it implies some kind of equality in the bargaining process. Assumption (4) looks difficult but it can be more simply stated: it says that taking away possible outcomes that were not chosen should not change the chosen outcome. Stated in this form it appears innocuous. It is, however, quite important to what follows.

7 Which means that if the function $u(.)$ represents the preferences of one of the bargainers, then so does any other function $a + bu(.)$ (where $b$ is positive), and that no other function represents the preferences.
The idea is the following. These rules are presented to the bargainers. If they agree to them as general principles, then they would be happy with the implied outcome.

What is the implied outcome? Nash showed something very clever: if these assumptions are accepted then the implied outcome is that outcome which maximises the product of the utility functions of the two bargainers. To prove this result would take us beyond the scope of this book – but we can apply it. Let us apply it in the context of the bargaining problem between the monopolist and the monopsonist. To make life simple let us consider a very simple case – you can then try and extend it.

Consider a monopolist with the marginal cost schedule pictured in the figure below. This is the upward-sloping line, which would be the firm’s supply function if it were a competitive firm. Suppose the monopsonist has the downward sloping marginal revenue curve in the figure – which would be the firm’s demand function if it were a competitive firm. If there was perfect competition, the equilibrium price would be 50 and the quantity exchanged would be 50.

The bargaining is over the price. Given the chosen price, the lesser of the demand and supply determines the quantity traded. The figure shows what would happen if the price agreed was 25. The monopolist would supply 25 units and this would be the amount traded. The monopsonist would have a surplus equal to the blue area – that is 1562.5 – and the monopolist would have a surplus equal to the red area – that is 312.5.

Let us assume that both the bargainers are risk-neutral and therefore their utilities are equal to their surpluses. At what price would the product of their surpluses (and hence the product of their utilities) be maximised? Take a price $p$ above the equilibrium – as in the figure above. The quantity exchanged would be $p$ (given by the supply curve) and the surplus of the seller would be $p^2/2$. The surplus of the buyer would be the equivalent of the blue shaded area – namely $p^2/2 + p(100-2p)$. The product of the two surpluses would be thus

$$[p^2/2][p^2/2 + p(100-2p)] = p^4(200 - 3p)/4 = 50p^3 - 3p^4/3.$$
If we maximise this with respect to \( p \) we find that the product-maximising value of \( p \) is \( 50^9 \). If we do the same with a price greater than 50, we get the same solution. Hence the outcome of the bargaining process if the bargainers agree to the Nash bargaining solution is the competitive price and hence the competitive outcome. Your intuition is confirmed!

This is a nice result and you should investigate how general it is. You can take different demand and supply schedules and repeat the analysis. You can probably guess that it is not general: we know that the competitive equilibrium is the outcome which maximises the sum of the surpluses, while the Nash solution to the bargaining problem is the outcome which maximises the product of the surpluses. These may be, but need not be the same. But it is reassuring in this simple context that we get this nice result.

You should also note that the assumptions underlying the Nash solution are plausible but not necessarily true. In addition, to apply the solution in practice requires the two bargainers to agree to the assumptions. You might like to ask if this is the procedure adopted when real bargaining takes place. In some instances it is – for example, when both sides to a bargaining dispute agree to hand over the problem to an independent tribunal, which has agreed terms of referral. Independent pay review bodies are good examples of this kind of tribunal. But in other cases, the bargaining breaks down without such a referral, and the two sides resort to industrial action of various kinds (such as strikes and lockouts). It would be interesting to examine such cases, but that would take us beyond the scope of this book.

28.11: Mathematical Appendix

We first provide a general proof of the profit-maximising condition for a monopolist. This is short and simple. We first note that profits are the difference between revenue and costs, and that both of these are functions of output. We denote revenue by \( R(y) \) and hence get:

\[
\text{Profit} = \pi = R(y) - C(y)
\]

Applying the usual condition for a maximum, that \( d\pi/dy = 0 \) we get immediately that

\[
d\pi/dy = dR(y)/dy - dC(y)/dy = 0
\]

and hence that

\[
dR(y)/dy = dC(y)/dy
\]

which simply says that marginal revenue should be equated with marginal cost.

We then invoke the second-order condition, which says that the slope of \( \pi \) should be decreasing at the maximum; that is, that \( d^2\pi/dy^2 \) should be negative. This yields the condition that

\[
d^2R(y)/dy^2 < d^2C(y)/dy^2
\]

This says that the marginal cost curve should cut the marginal revenue curve from below.

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9 If we denote the product by \( P \) then \( dP/dp = 150p^2 - 3p^4 \), which, when equated to zero gives \( p = 50 \) (and \( p = 0 \), which is a minimum).
We now provide a general proof of the profit-maximising condition for a monopsonist. This too is short and simple. Indeed it is almost identical to the derivation of the profit-maximising condition for a monopolist. We first note that profits are the difference between revenue and costs, and that both of these are functions of the quantity of labour purchased. We denote revenue by $R(l)$ and costs by $C(l)$ and hence get:

$$ \text{Profit} = \pi = R(l) - C(l) $$

Applying the usual condition for a maximum, that \( d\pi/dl = 0 \) we get immediately that

$$ d\pi/dl = dR(l)/dl - dC(l)/dl = 0 $$

and hence that

$$ dR(l)/dl = dC(l)/dl $$

which simply says that the value of the marginal product of labour (the marginal revenue from labour) should be equated with the marginal cost of labour – as stated in the text.

We then invoke the second-order condition, which says that the slope of \( \pi \) should be decreasing at the maximum; that is, that \( d^2 \pi/dl^2 \) should be negative. This yields the condition that

$$ d^2 R(l)/dl^2 < d^2 C(l)/dl^2 $$

This says that the marginal cost of labour curve should cut the value of the marginal product of labour curve from below.