Chapter 20: Intertemporal Choice

20.1: Introduction

We are now in a position to apply our methodology in a variety of contexts, including two particularly important ones – intertemporal choice and risky choice. As we will see, we can use the apparatus we have constructed to analyse these interesting problems.

We start with intertemporal choice. This is choice through time. Many of the important problems that we face are intertemporal choice problems – including possibly the most important, that of choosing an optimal consumption stream through time. We should emphasise the importance of the problem of these problems in economics. While our analysis so far has been static, many of the important decisions in life involve choices whose consequences will occur in the future. The most obvious is the decision how to allocate ones income through time: whether to save for retirement; whether to build up a pension fund; whether to save for a holiday. Equally important, though not specifically mentioned in this chapter are problems faced by firms: whether to invest in new technology; whether to hold inventories; whether to borrow money in the capital market. This chapter discusses how such decisions might be taken, and emphasises the important role of the rate of interest in such decisions. A key variable manipulated by governments is the rate of interest: by lowering it they hope to encourage investment; by raising it they hope to encourage saving. In this chapter we will see how these things work.

In reality many of these problems involve many periods of time, but we can do all the analysis necessary with just two time periods, periods 1 and 2. There is no need for these periods to be the same size, so, if you like, you can think of them as ‘the present’ (extending a certain distance into the future) and ‘the future’ (extending the rest of time). For the time being we do not want to get involved with problems of uncertainty (that is our next topic) so we will assume that there is no uncertainty involved with defining or thinking about these two periods.

Let us consider the problem of optimal consumption through time in this two-period world. We shall keep things very simple and assume that the individual gets some income in each of these two periods and has to decide when to consume that income. We shall assume that there is a capital market in which the individual can borrow and save – so if the individual wants to consume more than his or her income in the first period, then he or she borrows some money in this capital market and repays it plus interest in period 2. Alternatively if he or she wants to consume more than his or her income in period 2 then he or she invests a part of his first period income in the capital market, realising the proceeds and the interest in the second period.

We shall make things very simple by assuming a perfect capital market – which is one in which he or she can freely borrow and save at a constant an fixed rate of interest. In a concluding section we shall discuss how things change if the capital market is imperfect. This latter case is probably empirically more relevant, but the crucial parts of the analysis can be done by assuming a perfect capital market.

20.2: The Intertemporal Budget Constraint
We denote by $m_1$ and $m_2$ the income of the individual in the two periods. We denote by $c_1$ and $c_2$ the consumption in the two periods. We assume that utility is a function of $c_1$ and $c_2$ – later we will discuss what form this may take. Further, as discussed above, we assume a perfect capital market with a constant interest rate $r$ – denoted by $r$.

What is the budget constraint in this world? Well obviously it passes through the point $(y_1, y_2)$ as the individual can always choose simply to consume his or her income each period. But, suppose the individual would prefer an alternative pattern of consumption, what possibilities are open to him or her?

If the individual wanted to consume nothing in period 1 then he or she could save the income $m_1$, investing it at the rate of interest $r$, earning interest $rm_1$ and thus having $m_1(1+r)$ plus his or her income $m_2$ to spend in period 2. This would give a maximum consumption in period 2 of $m_1(1+r) + m_2$. Alternatively if he or she wanted to consume nothing in period 2, then in period 1 he or she could spend his income of that period, $m_1$, plus what he or she could borrow on the strength of his or her income $m_2$ to spend in period 2. Now the amount that could be borrowed on this basis is $m_2/(1+r)$ since this would have become $m_2$ with interest at $r$ added by period 2. So the maximum consumption in period 1 would be $m_1 + m_2/(1+r)$. More generally if he or she borrows an amount $c_1 - m_1$ then he or she would have to pay back in period 2 this amount plus interest - that is, an amount $(c_1 - m_1)(1+r)$. So the difference $m_2 - c_2$ would have to equal this amount. So the budget constraint is given by $(c_1 - m_1)(1+r) = m_2 - c_2$ which we can write as

$$c_1(1+r) + c_2 = m_1(1+r) + m_2$$

(20.1)

which simply says that the future value of the consumption stream must equal the future value of the income stream. Alternatively we can write the budget constraint (dividing both sides by $(1+r)$) as

$$c_1 + c_2/(1+r) = m_1 + m_2/(1+r)$$

(20.2)

which simply says that the present value of the consumption stream must equal the present value of the income stream. Graphed in $(c_1, c_2)$ space this is obviously a straight line with slope $-(1+r)$. The higher is the rate of interest the steeper is this line (in magnitude). As the rate of interest rises, the budget line rotates clockwise around the initial income point: it becomes steeper. (Compare the budget lines in figures 20.2 – an interest rate of 20% - and that in figure 20.3 – a rate of interest 0%). Obviously if $r$ is zero it has slope -1.

In figure 20.2 we give an example with $r = .2$ (that is, a rate of interest 20%) and with incomes $m_1 = 30$ and $m_2 = 50$. In this figure the amount of consumption in period 1 is on the horizontal axis and the amount of consumption in period 2 on the vertical axis. Note that the budget line passes through the point $(30, 50)$ as the individual can always simply consume his or her initial incomes. If the individual wanted to consume everything in the first period he or she could consume 30 (his or her income of the first period) plus $50/1.2 = 41.66$ (the amount he or she could borrow on the strength of paying it back plus interest of 20% with the 50 income of period 2) which is equal to 71.66 – the intercept on the horizontal axis. If the individual wanted to consume everything in the second period he or she could consume $30 \times 1.2 = 36$ (the income of the first period plus interest at 20%) plus 50 (the income of the second period) which is equal to 86 – the intercept on the vertical axis.

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1 If the rate of interest is 10% then $r = 0.1$; if the rate of interest if 20% then $r = 0.2$; and so on.
We see that the slope of this budget constraint is \(-\frac{86}{71\frac{2}{3}} = -1.2\) – that is, minus one plus the rate of interest.

We will later find it useful to introduce some new terminology – the rate of return on an investment. This is simply one plus the rate of interest – it tells us how much money we get back in period 2 for every 1 unit saved in period 1.

**20.3: Optimal Intertemporal Choice**

We have found the intertemporal budget constraint. To find the optimal choice we now need to consider the preferences of the individual over bundles of intertemporal consumption \((c_1, c_2)\). In chapter 21 we will consider a particular type of preferences which seems to have good empirical validity. In the meantime, in this chapter, we shall consider some more general preferences.

Suppose we were trying to construct the indifference map in \((c_1, c_2)\) space, what properties would it possess? First we would expect it to be convex: as consumption in period 2 decreased we would expect that the compensation required in period 1 to increase; as consumption in period 1 decreased we would expect that the compensation required in period 2 to increase. You should be clear about this: suppose that with consumption of 50 in each of the two periods, the individual would be happy to give up 1 unit of consumption in period 1 if he or she were compensated with 1.2 extra units of consumption in period 2; we might expect that when the consumption is just 10 in period 1 but 90 in period 2, that the individual would want more than 1.2 units of extra period 2 consumption to be compensated for giving up one of the 10 period 1 units. What else? Might we expect a symmetrical map? Well, this depends on whether the individual treats the two periods symmetrically: if he or she did we might get something like figure 20.3 – in which we can analyse the optimal choice as well as how that changes if the rate of interest changes. Note, however, that symmetry is unlikely - most people appear to have a preference for consumption in the present over consumption in the future.
In figure 20.3 the rate of interest assumed is obviously zero – the slope of the budget line is \(-1\). So for every 1 unit less of consumption in period 1 the individual can – through lending - consume 1 more unit in period 2, and for every 1 unit less of consumption in period 2 the individual can – through borrowing - consume 1 more unit in period 1. The initial endowment point (30 units in period 1 and 50 units in period 2) is indicated with an X. The optimal point – on the highest indifference curve consistent with the budget constraint – is indicated with an asterisk. Because of the symmetry of the preferences, and the fact that the rate of interest is zero, this optimal point is symmetrical – with 40 units of consumption in period 1 and 40 units in period 2. The individual uses the capital market to even out his or her consumption. In period 1 he or she borrows 10 units in the capital market and consumes his or her income in period 1 plus that borrowing to give a total consumption of 40. In period 2 he or she pays back this 10 (with no interest in this case because the rate of interest is zero) and therefore consumes 10 less than his or her income in period 2. This individual in this situation is a borrower.

Now consider what happens when the rate of interest rises. This is an exercise that you can do yourself. When the rate of interest rises the budget constraint rotates around the point X. The optimal point clearly moves to the left – and so the individual consumes less in period 1 and thus borrows less.

You might like to consider whether this individual might become a saver. The answer is ‘yes’ – if the optimal point moves to the left of the initial point X. At what rate of interest will this happen? This depends upon the slope of the indifference curve at the point X. For reasons which will become obvious let us suppose that this slope\(^2\) is \(-1-s\). Then the individual switches from being a borrower to being a saver when the interest rate is higher\(^3\) than \(s\). So we have the nice result that this individual is a borrower at low rates of interest, reduces the extent of his or her borrowing as the interest rate rises, and becomes a saver at sufficiently high rates of interest. As you might be able to work out, this is a fairly general result, though obviously it depends upon the preferences of the individual. For the particularly preferences portrayed in figure 20.3 we have the relationship between saving and the rate of interest shown in figure 20.4.

\(^2\) For example if the slope there is \(-1.6\) then \(s = 0.6\) (that is, 60%).

\(^3\) Recall that the slope of the budget line is \(-1+r\) so the budget line is tangential to the indifference curve passing through the point X if and only if \(r = s\).
The curve that starts out positive, becomes zero around a rate of return 1.68 (a rate of interest = 0.68 that is 68%) and then becomes negative, is the net demand for consumption in period 1. If it is positive it is the amount borrowed in period 1 and if it is negative it is the amount saved in period 1. The other curve is the net demand for consumption in period 2. If it is negative it is the amount repaid in period 2 (including the interest) and if it positive it is the total return from saving in the first period. Obviously these two curves cross the axis at the same point: if the individual borrows in the first period he or she has to repay in the second; if he or she saves in the first period there is a positive return in the second.

The initial endowment point affects the behaviour of the individual. If, for example, one of the two incomes were to change then the borrowing/saving behaviour will change. For example if the income in the first period were to increase, we get the relationship between the net demands and the first period income shown in figure 20.6. (In this we keep the second period income fixed at 50 and the rate of interest fixed at 20%.

The individual stops being a borrower and becomes a saver if his or her first period income becomes sufficiently high. (The line that starts out positive and then becomes negative is the net demand for consumption in period 1; the other is the net demand for consumption in period 2.)

The shape of these net demand curves depend upon the preferences of the individual. So far we have assumed symmetrical preferences but it may well be empirically more reasonable to assume asymmetrical preferences of a type where more weight is put on present consumption. We shall discuss this in more detail in the next chapter – but in the meantime let us present a case with more weight on present consumption and compare it with the results presented above. Compare first figure 20.9 with figure 20.3.
Note that this is no longer symmetrical and more weight is put on first period consumption. If we carry out the same comparative static exercise that was portrayed in figure 20.4 with these new preferences we get figure 20.10.

Compared with figure 20.4 we get much more borrowing than before. This is the – perhaps obvious – consequence of the greater weight on first period consumption.

20.4: Present and Future Values and Discounting

If we are in a two-period world and we have incomes \( m_1 \) and \( m_2 \) in the two periods, and we have a rate of interest \( r \), then the future value of this stream – that is the value in period 2 – is given by

\[
m_1(1 + r) + m_2
\]

because the income of \( m_1 \) in the first period grows to \( m_1(1 + r) \) with the interest.

The present value of this income stream – that is the value in period 1 – is given by

\[
m_1 + m_2/(1+r)
\]

because \( m_2/(1+r) \) is what can be borrowed on the strength of being able to repay \( m_2 \) in period 2.

If we live in a \( T \)-period world with income \( m_t \) in period \( t \) (\( t = 1, 2, ..., T \)) then with interest \( m_1 \) in period 1 grows to \( m_1(1+r)^{T-1} \) by period \( T \), \( m_2 \) in period 2 grows to \( m_2(1+r)^{T-2} \) by period \( T \), etc, so the future value of this stream – that is the value in period \( T \) - is given by

\[
m_1(1+r)^{T-1} + m_2(1+r)^{T-2} + ... + m_{T-1}(1+r) + m_T
\]

The present value of this income stream is
because \( m_2/(1+r) \) is what can be borrowed in period 1 on the strength of being able to repay \( m_2 \) in period 2, ..., \( m_{T-1}/(1+r)^{T-2} \) is what can be borrowed in period 1 on the strength of being able to repay \( m_{T-2} \) in period \( T-1 \), and \( m_T/(1+r)^{T-1} \) is what can be borrowed in period 1 on the strength of being able to repay \( m_T \) in period \( T \). We call \( m_2/(1+r) \) the discounted value of income \( m_2 \) in period 2, ..., \( m_{T-1}/(1+r)^{T-2} \) the discounted value of income \( m_{T-1} \) in period \( T-1 \), and \( m_T/(1+r)^{T-1} \) the discounted value of income \( m_T \) in period \( T \). We say that these incomes are being discounted at the market rate of interest \( r \).

### 20.5: Imperfect Capital Markets

You may well object to the assumption that the individual can freely borrow and lend at the market rate of interest. You might reasonably point out that usually one pays a higher rate of interest when borrowing than when saving. If this is the case, it is what we term an imperfect capital market. What does it imply for the budget constraint? When the individual is borrowing – that is when he or she wants to move rightwards and downwards from the initial endowment point – the rate of interest is higher than when the individual is saving - – that is when he or she wants to move leftwards and upwards from the initial endowment point. This implies that the magnitude of the slope of the budget line is greater for movements to the right and down than for movements to the left and up. It follows that there is a kink at the initial endowment point. An example is given in figure 20.18, in which the initial incomes are 30 and 50 (as in the examples above) and in which the rate of interest on saving is 10% while that on borrowing is 50%. (We have chosen a deliberately exaggerated example to get a figure which illustrates clearly the point that we want to make.) The budget line now has a kink at the initial income point, and two different slopes: -1.5 below the kink point and -1.1 above it. The blue dotted line is the budget constraint that would exist in a perfect capital market with a rate of interest of 30% on both saving and borrowing. You might realise the possible impact of the kink point on behaviour – it is quite likely that the optimal point will be at the kink point (if the marginal rate of substitution between period 1 and period 2 consumption is between 1.0 and 1.4) – so that the individual will neither borrow or save.

20.7: Summary

*The budget line with a fixed rate of interest \( r \) has slope \(-(1 + r)\).*
Whether the individual is a borrower or a lender depends on the individual's preferences, on the initial endowment and the rate of interest.

Increases in the rate of interest (usually) cause a borrower to reduce the level of borrowing.

Decreases in the rate of interest (usually) cause a saver to reduce the level of saving.

In an imperfect capital market there is a kink in the budget constraint at the endowment point, with a steeper section below the kink point and a flatter section above it.