Testing Choquet expected utility

Lukas Mangelsdorff a, Martin Weber b, *, 1

a Fakultät für Wirtschafts- und Sozialwissenschaften, Universität Kiel, Olshausenstrasse 40, 24098 Kiel, Germany
b Fakultät für Betriebswirtschaftslehre, Universität Mannheim, I. 5.2, 68131 Mannheim, Germany

Received August 1992, final version received October 1993

Abstract

Many theories have been developed to model decision behavior under ambiguity. In this paper we empirically investigate Choquet expected utility, which is based on nonadditive probabilities. We first replicate Ellsberg paradox behavior. Then we elicit nonadditive probabilities (called capacities) for individual decision makers. Those capacities did not have all properties theoretically required by CEU. Finally we found that CEU is not really superior to EU in predicting the participants' decisions.

Keywords: Ellsberg paradox; Ambiguity; Choquet expected utility; Nonadditive probabilities; Experimental economics

JEL classification: D81

1. Introduction

Consider a lottery where you win if a white ball is drawn from an urn which contains exactly 50 white and 50 yellow balls. Alternatively, consider a second urn that contains 100 white or yellow balls in an unknown proportion. You are allowed to pick the color you want to win. If the amount of winning is the same

* Corresponding author.
1 We would like to thank Graham Loomes, Rakesh Sarin, and an anonymous referee for their helpful comments on former versions of the paper. The work was supported by grant We993/5-1 of the Deutsche Forschungsgemeinschaft. Funds for paying the participants were provided by the Graduiertenkolleg für Betriebswirtschaft of the University of Kiel.
for both urns a majority of people will prefer to bet on the first urn over the second — they dislike being unsure about the probability of winning in the second urn. However, following subjective expected utility (SEU) theory, subjects should be indifferent between both urns or even prefer the second urn if they believe that one color is more likely than the other.

Decision situations where subjects are unsure about the probabilities are also referred to as decision making under ambiguity. The fact that aversion towards ambiguity can affect preferences was first highlighted by Ellsberg (1961). He showed that ambiguity aversion leads to a violation of Savage's sure-thing principle (Savage 1954) and thus cannot be described by SEU.

Quite a few empirical investigations have demonstrated that decision makers can have a nonneutral attitude towards ambiguity. Many theories have been proposed to model attitudes towards ambiguity. For an overview on recent developments in decision making under ambiguity see Camerer and Weber (1992).

One notable approach to modeling decision making under ambiguity makes use of the concept of nonadditive probabilities, called capacities. Schmeidler (1989) introduced the Choquet expected utility (CEU) theory, which makes use of the concept of capacities and is able to model attitudes towards ambiguity. In this paper we will experimentally test Choquet expected utility theory. We will compare different ways to elicit nonadditive probabilities, test implications of the theory, and try to predict decision makers' behavior in situations that involve decisions under ambiguity.

In Section 2 we give a brief description of Choquet expected utility, and we explain in detail ways to elicit capacities. In Section 3 we state the hypotheses and describe the experimental design. The results are presented in Section 4. As we got some striking results that could be due to an order effect in our design we replicated parts of the study after changing the design. These results are presented in Section 5.

2. Choquet expected utility (CEU)

2.1. The theory

By relaxing the independence axiom of Anscombe and Aumann (1963) in order to hold for comonotonic acts only Schmeidler (1989) derives a utility theory that allows for nonadditive probabilities. Two acts (alternatives) are comonotone if the outcomes of the two acts under different states do not move in opposite directions. More formally, acts \( f \) and \( g \) are comonotone if there are no states \( s \) and \( t \), with

\[ \text{Other theoretical work developing CEU theories includes Gilboa (1987), Sarin and Wakker (1992), Schmeidler (1986), Wakker (1989), and Nakamura (1990).} \]
f(s) > f(t) while g(s) < g(t). An act f is preferred to an act g if the Choquet expected utility for f is larger than for g; i.e., CEU(f) > CEU(g). For a finite set of states CEU(f) is defined by

\[
CEU(f) = u_1 \pi(s_1) + \sum_{i=2}^{n} u_i \left[ \pi \left( \bigcup_{j=1}^{i} s_j \right) - \pi \left( \bigcup_{j=1}^{i-1} s_j \right) \right],
\]

with

\[
u_i = u[f(s_i)], u_1 \geq u_2 \geq \ldots \geq u_n,
\]

where u is a real valued utility function of the outcomes, and \( \pi \) is a capacity with the following properties:

- \( \pi(S) = 1 \) where S is the set of all states
- \( \pi(\emptyset) = 0 \)
- \( \pi(s) \leq \pi(t) \) if \( s \subseteq t \).

A capacity is nonadditive: the capacity of the union of two disjoint events is not necessarily equal to the sum of the capacities of the events.

When calculating Choquet expected utility of an act, one first has to rank the different states \( s_i \) according to their attractiveness. Note that the ranking of states could differ from lottery to lottery. In CEU the utility of an outcome under a state \( u_j \) is weighted with something one might call difference of transformed cumulative probabilities. In case of additive capacities (i.e., probabilities), CEU reduces to (subjective) expected utility. As CEU contains expected utility as a special case, u has to be the traditional utility function.

2.2. Capacities

In our empirical study we considered two decision problems: the three-color Ellsberg urn, and a problem where subjects had to bet on the share price of the Japanese Dai Ichi Kangyo Bank. As this company is not well known among German students, we expected a high ambiguity aversion for this decision problem. We will show how to elicit the capacities for both decision problems. First, we will consider the Dai Ichi Kangyo Bank case, as there are two different ways to elicit the capacities. Later we will concentrate on this case. The Ellsberg case is slightly more complex since there are more events for which the capacities have to be determined. We will assume that the decision maker's utility function has already been assessed.

2.2.1. Dai Ichi Kangyo Bank

In the Dai Ichi Kangyo Bank case the participants had to choose between four pairs of lotteries. Each pair consisted of one lottery defined on the share price of

\footnote{Heath and Tversky (1991) as well as Keppe and Weber (1994) found that ambiguity aversion is related to the judged knowledge of the event.}
the Dai Ichi Kangyo Bank and of one lottery with given probabilities. The following example may serve as an illustration:

**Lottery I:** You will get DM 50 if on the 25 November 1991, at Tokyo stock exchange, the price of a share of Dai Ichi Kangyo Bank, one of the world’s ten largest banks, was higher than (>) Yen 2600. Otherwise you will get nothing.

**Lottery J:** You will get DM 50 if a black ball is drawn out of an urn containing 50 black and 50 white balls. Otherwise you will get nothing.

Calculating CEU(I) and CEU(J), we get:

\[
\text{CEU}(I) = u(50) \pi(>)
\]

\[
\text{CEU}(J) = u(50) \frac{1}{2}
\]

\(\pi(>)\) gives the capacity for the event that the price of Dai Ichi is above the given limit; \(\pi(\leq)\) gives the capacity for the event that the price is below or equal the limit.

If the decision maker is indifferent between I and J the capacity \(\pi(>)\) is equal to .5 since (2) and (3) have to be equal. If the decision maker prefers I to J, \(\pi(>)\) is greater than .5, thus representing ambiguity seeking. If J is preferred to I, \(\pi(>)\) is smaller than .5 thus showing ambiguity aversion. Numerical values for the capacities can be calculated from indifference statements derived by changing one lottery. A lottery can be changed by changing the chance of winning or by changing the amount to be won. We will first demonstrate how a capacity can be calculated if the chance of winning is changed.

**Changing chances of winning.** If I is preferred to J the decision maker is asked how many black balls have to be added to make him or her indifferent between I and J. Taking \(X\) as the number of black balls that have to be added \(\pi(>)\) is given by the following equation:

\[
\pi(>) = \frac{50 + X}{100 + X}
\]

(4)

Things are very much similar if a subject prefers J over I. Taking \(X\) as the number of black balls that have to be removed \(\pi(>)\) is then given by the following formula:

\[
\pi(>) = \frac{50 - X}{100 - X}
\]

(5)

---

4 The experiment was done on 29 November 1991.

5 For nonambiguous events we set the capacities to be equal to the relative likelihoods.

6 The participants were explicitly told that adding black balls (or removing black balls) does not mean that they substitute white balls (or that they are substituted by white balls). Thus a change of the number of black balls also leads to a change of the total number of black and white balls. One could think of other devices to elicit the probability equivalents; e.g., a probability wheel.
Changing amount to be won. Instead of changing the chances of winning we can also change the DM amounts subjects can win ('DM statements'). If I is preferred over J, Lottery J has to be made more attractive. This can be done by increasing the amount of money to be won if a black ball is drawn. The individual is asked how much money he or she must receive (= 50 + Y DM) if a black ball is drawn in order to be indifferent between this new Lottery J' and I. \( \pi(>) \) is determined by the following equation:

\[
\pi(>) = 0.5 \frac{u(50 + Y)}{u(50)}
\]

In the case J is preferred over I, the amount to be won if a black ball is drawn has to be reduced, thus Y in (6) is negative.

Comparing both methods to elicit capacities, changing the amount of winning seems to be easier than changing the probability to win. However, changing probabilities is a direct assessment of the capacities independent of any utility function. The necessity of using utility values to calculate the capacities can introduce additional error.

### 2.2.2. Ellsberg

Ellsberg (1961) considered an urn that contained 30 black balls and 60 red or yellow balls where the proportion of red and yellow balls is unknown. Four lotteries are defined in Table 1. Lottery C, for example, pays 50 DM if a black ball or a yellow ball is drawn from the urn.\(^7\)

For three elementary events in Ellsberg's three-color problem (drawing a black, a red, and a yellow ball), there are \(2^3 = 8\) events for which the capacities have to be determined. As the capacities do not need to be additive, the determination gets more complicated than in the case of additive probabilities. The capacity

---

\(^7\)When having to choose between A and B, most people prefer Lottery A over Lottery B. Now both lotteries are varied in the same way. One may also win DM 50 if a yellow ball is drawn from the urn (Lotteries C and D). According to Savage's sure thing principle (Savage 1954) this alteration should not lead to a change in preferences. However, most people prefer Lottery D over Lottery C, a clear violation of subjective expected utility theory.
\(\pi(\text{BLACK} \cup \text{YELLOW})\), for example, can no longer be determined by adding \(\pi(\text{BLACK})\) and \(\pi(\text{YELLOW})\).

For some of the eight events the capacities are straightforward (Schmeidler 1989, pp. 571). For nonambiguous events specially the capacities should be equal to the relative likelihoods:
- \(\pi(\emptyset) = 0\)
- \(\pi(\text{BLACK}) = 1/3\)
- \(\pi(\text{YELLOW} \cup \text{RED}) = 2/3\)
- \(\pi(\text{BLACK} \cup \text{YELLOW} \cup \text{RED}) = 1\).

In addition we assume the capacities to be identical for symmetric events \(^8\):
- \(\pi(\text{RED}) = \pi(\text{YELLOW})\)
- \(\pi(\text{BLACK} \cup \text{RED}) = \pi(\text{BLACK} \cup \text{YELLOW})\).

As an example of how to calculate capacities, we will demonstrate how to calculate \(\pi(\text{BLACK} \cup \text{YELLOW})\) assuming that \(\pi(\text{YELLOW})\) had been previously determined. The decision maker is asked to choose between Lotteries C and D (see Table 1). Calculating CEU(C) and CEU(D) we get:

\[
\begin{align*}
\text{CEU}(C) &= u(50) \pi(\text{BLACK}) + u(50) \left[ \pi(\text{BLACK} \cup \text{YELLOW}) - \pi(\text{BLACK}) \right] \\
&= u(50) \pi(\text{BLACK} \cup \text{YELLOW}) \quad (7)
\end{align*}
\]

\[
\begin{align*}
\text{CEU}(D) &= u(50) \pi(\text{RED}) + u(50) \left[ \pi(\text{RED} \cup \text{YELLOW}) - \pi(\text{RED}) \right] \\
&= u(50) \frac{2}{3} \quad (8)
\end{align*}
\]

If the decision maker is indifferent between C and D the capacity \(\pi(\text{BLACK} \cup \text{YELLOW})\) is equal to 2/3. If the decision maker prefers C or D, one lottery has to be changed to derive an indifference statement. In the Ellsberg case the capacities can only be determined by changing the amount to be won. A change in the number of balls (i.e., a change in the chances of winning), leads to different ambiguous or unambiguous events. The event is, for example, no longer ‘drawing a black ball out of the original Ellsberg urn’ but ‘drawing a black ball out of the changed urn.’

If C is preferred to D, Lottery C has to be made less attractive. This can be done by reducing the amount of money to be won if a black ball is drawn. The individual is asked how much money he or she must receive (= 50 - Y DM) if a

---

\(^8\) In this setting we assume that subjects have no color preference.
black ball is drawn in order to be indifferent between this new Lottery C' and D. As 50 - Y is smaller than 50, a change in the ranking of desirability of the states occurs. We get:

\[
\text{CEU}(C') = u(50) \pi(\text{YELLOW})
\]
\[
+ u(50 - Y) \left[ \pi(\text{BLACK} \cup \text{YELLOW}) - \pi(\text{YELLOW}) \right]
\]

In case of indifference between C' and D, (8) is equal to (9). We get:

\[
\pi(\text{BLACK} \cup \text{YELLOW}) = \pi(\text{YELLOW})
\]
\[
+ \frac{u(50)}{u(50 - Y)} \left[ \frac{2}{3} - \pi(\text{YELLOW}) \right]
\]  

(10)

In case D is preferred to C, the amount to be won on black has to be increased until the decision maker is indifferent between the new Lottery C" and Lottery D. At the point of indifference the amount to be won on black in C" (= 50 + Z) is greater than 50. We get:

\[
\text{CEU}(C'') = u(50 + Z) \pi(\text{BLACK})
\]
\[
+ u(50) \left[ \pi(\text{BLACK} \cup \text{YELLOW}) - \pi(\text{BLACK}) \right]
\]

In case of indifference between C" and D, (11) is equal to (9). We get:

\[
\pi(\text{BLACK} \cup \text{YELLOW}) = 1 - \frac{1}{3} \frac{u(50 + Z)}{u(50)}
\]  

(12)

The other capacities of the Ellsberg case can be derived similarly.

3. Hypotheses and experimental design

3.1. Methods for testing CEU

A variety of approaches can be used to test a preference theory (Weber and Camerer 1987). On a basic level, one could investigate if the theory (especially some key axioms) in principle is able to model certain preference patterns. This is how Ellsberg suggests SEU to be tested. Analogously, in a risky setting, quite a few researchers have used this approach to find out which of the recent generalizations of expected utility (EU) theory are best able to fit certain behavioral patterns (Harless and Camerer 1993 for an overview).

Using a different approach, one can test some important implications of the theory. One implication of standard prospect theory (Kahneman and Tversky 1979) is, for example, that the decision weighting function is identical for gains
and losses. As Currim and Sarin (1989) show, this is empirically not true. In their recent extension of prospect theory Tversky and Kahneman (1992) allow for different decision weight functions for the gain and the loss domains.

Finally, as most of the recent generalizations of EU are proposed in order to describe behavior, one could test this claim. This is definitely important for applications in an area like marketing. Here one does not want a theory that in principle is able to model behavior but that — after assessing the individual parameters — does not predict actual preferences. Note that a complex theory, which in principle is able to predict a richer set of preferences, is not necessarily more accurate than a simpler theory.

It is important to distinguish between the different approaches because they imply different ways of setting up the experiments. For the first two approaches it is often sufficient to collect ordinal data, which in general show less error. For the third approach, utility functions or decision weights (i.e., cardinal data), have to be elicited. Complex theories often have more parameters to allow for a richer set of preferences. The error in assessing these additional parameters can make the complex theories worse than simple theories in predicting preference judgments.°

3.2. Hypotheses

Our study will focus on the second and third approaches described above: we want to test certain implications of Choquet expected utility experimentally. In addition, we wanted to test the predictive power of the theory.

From previous empirical research, we know that decision makers show some degree of ambiguity aversion when confronted with Ellsberg’s three-color problem and with a lottery based on something like the Dai Ichi Kangyo Bank’s stock price. We want to investigate if this ambiguity aversion is reflected in the capacities, and if it is, how strong the ambiguity effect is. As a baseline we will compare the capacities to probabilities that are derived by the principle of insufficient reason.

Hypothesis 1.

Capacities for ambiguous events differ from probabilities that could have been derived according to the principle of insufficient reason.

CEU theory does not distinguish between different methods of eliciting capacities. In the assessment of utility functions it is well known that different assessment procedures could yield significantly different utility functions (Hershey, Kunreuther and Schoemaker 1982, Hershey and Schoemaker 1985). It seems

° It is also worthwhile to model and to test the implications of preference theories in specific economic settings (i.e., markets). Sarin and Weber (1993), for example, showed that ambiguity effects also persisted in an experimental market setting.
therefore worthwhile to test if both procedures to elicit capacities described in Section 2 will yield the same result.

**Hypothesis 2.**

Capacities elicited on the basis of chances of winning are the same as capacities elicited on the basis of the amount to be won.

CEU does not distinguish between gain and loss domains. We formulate Hypothesis 3 because there is evidence that subjects have different degrees of ambiguity aversion in both domains (see Camerer and Weber 1992).

**Hypothesis 3.**

Capacities are identical when derived from gain or loss statements.

Finally, we will investigate if CEU is able to predict the intuitive preferences in decision situations with ambiguity. We use EU with probabilities derived by the principle of insufficient reason as a baseline for comparison.

**Hypothesis 4.**

CEU is better able to predict subjects’ decisions than expected utility.

### 3.3. Experimental design

In the first part of the questionnaire (the elicitation sample) we asked questions to derive the individual parameters (capacities and utility function). Based on these parameters, we tested Hypotheses 1–3. Then we presented some more alternatives (the validation sample) from which we tried to predict the evaluation. We used two different sets of events (the Ellsberg case and Dai Ichi Kangyo Bank case).

**Elicitation sample.** In the Dai Ichi Kangyo Bank case the subjects had to choose between four pairs of Lotteries (I–J as shown above, K–L, M–N, and Q–P). In lottery pair K–L, Lottery K only differs from Lottery I insofar as one wins DM 50 if the share price was smaller than or equal to (≤) Yen 2600. L is identical to J. Lottery pairs M–N and P–Q are identical to pairs I–J and K–L; however, all amounts were negative. After each decision, subjects had to specify what changes had to be made in order for them to be indifferent between the lotteries. We asked for the number of balls to be changed and for the amount of money to be changed.

In the Ellsberg case, subjects had to choose between Lotteries A and B, and C and D (see Table 1). Then, the participants had to choose between two more pairs of lotteries: E and F, and G and H. Those were identical to the lottery pairs A–B and C–D; however, they involved losses rather than gains. After each comparison, the participants had to specify what changes in the amount to be won had to be made in order for them to be indifferent between the lotteries.

The subjects’ utility functions were determined using a midpoint splitting procedure. First a subject had to fill in the certainty equivalent, X, for a 50–50
Validation sample. The validation sample consisted of eight Ellsberg lotteries (numbers 1 to 8) and eight Dai Ichi Kangyo Bank lotteries (numbers 9 to 16) presented in Table 2. The table gives the amount one wins or loses if the event described obtains. If the event does not obtain one receives nothing. In Lottery 7, for example, one loses DM 100 if a black or yellow ball is drawn from the Ellsberg urn, otherwise one gets nothing.

For each lottery, subjects were asked for their certainty equivalent. The number elicited had to be positive or negative depending on whether the lottery had a positive or a negative expected value. We asked for the certainty equivalent as follows:

We are interested in how you evaluate the following lotteries compared to a sure payment. Please tell us the DM amount you must get with certainty so that you are indifferent between this amount and the lottery. If the lottery consists of negative outcomes, tell us the DM amount you would have to pay so that you are indifferent between paying this amount and the lottery.

3.4. Procedure

The experiment was run as a questionnaire. The eight pairs of lotteries in the elicitation sample were presented first. The pairs were ordered so that negative and positive amounts varied as well as the type of event. Then the utility functions were elicited. At the end of the questionnaire the validation sample was presented in some random order.

Seventy-four students answered the questionnaire. All students were master students in business or economics. They were guided through most parts of the questionnaire by the experimenter. They took on average 40 to 50 minutes to fill in the questionnaire. For their participation subjects were paid a flat fee of DM
Table 3
Ellsberg-paradox behavior

<table>
<thead>
<tr>
<th>Decision patterns</th>
<th>Number of people choosing the pattern</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>positive</td>
</tr>
<tr>
<td>SEU rational</td>
<td>24 (35%)</td>
</tr>
<tr>
<td>Ambiguity averse</td>
<td>34 (49%)</td>
</tr>
<tr>
<td>Ambiguity seeking</td>
<td>3 (4%)</td>
</tr>
<tr>
<td>Others</td>
<td>8 (12%)</td>
</tr>
</tbody>
</table>

15 We excluded five participants from the analysis. They either gave incomplete answers in various parts of the questionnaire or they were prepared to pay more for a lottery than its best possible consequence.

4. Results

4.1. Ellsberg-paradox behavior

We first wanted to check if our subjects show an ambiguity effect. Whether people behave in an Ellsberg-paradox manner can be analyzed by looking at the decision patterns of the lottery pairs A–B vs. C–D (positive amounts) and pairs E–F vs. G–H (negative amounts). A SEU-rational decision maker should prefer A and C (E and G), or B and D (F and H) or be indifferent. An ambiguity averse subject should prefer A and D (E and H); while an ambiguity seeker should prefer B and C (F and G). Those indifferent in one pair and indicating preference in the second pair are labeled ‘others’ in Table 3.

Table 3 shows, that for positive amounts, 54% show some effect towards ambiguity, mainly being ambiguity averse. Only 35% behave according to SEU. For negative amounts we see a slight increase in SEU-rational subjects and a strong increase in ambiguity-seeking subjects.

For the Dai Ichi Kangyo Bank case, let us consider the lottery pairs I–J and K–L. A preference for I and L, for J and K and two indifference statements can be explained by SEU 12. A preference for I and K shows ambiguity seeking; while a preference for J and L shows ambiguity aversion. Again, answers indicating indifference only once are labeled ‘others’. Table 3 shows that, for positive

---

11 We did not pay subjects according to their performance for the following reasons. First, this is standard practice in these types of experiments (see, e.g., Currim and Sarin 1989 or the empirical research on ambiguity reviewed in Camerer and Weber 1992). Second, we have problems using the Becker, DeGroot, and Marschak (1964) procedure for this study. Subjects who show ambiguity effects violate SEU. However, SEU (or EU) is needed to prove that BDM elicits the true certainty equivalents.
amounts only 25% behave according to SEU, whereas 70% show some ambiguity
effect, mostly indicating ambiguity aversion. For negative amounts, around 39% 
of our subjects show a decision pattern compatible with SEU, whereas 51% show 
some attitude towards ambiguity (2/3 ambiguity averse and 1/3 ambiguity 
seeking).

The results clearly demonstrate that subjects show ambiguity effects. They also 
indicate that their behavior is different for gains and losses. Taking the Ellsberg 
and the Dai Ichi Kangyo Bank case together we did a chi-square test of the 
equality of the distributions of the decision patterns for gains and losses. The 
empirical value, 53.56, is far higher than the theoretical value of 11.34 (α = .01, 
three degrees of freedom). Thus, there is much support for rejecting the null 
hypothesis that there is no difference in the attitudes towards ambiguity for gains 
and losses.

4.2. Calculation of capacities

Hypothesis 1. Hypothesis 1 states that the capacities should be different from the 
probabilities derived by using the principle of insufficient reason. As we argued 
before, we only need to calculate π(RED) and π(BLACK ∪ RED).

The following restrictions have to be fulfilled for a capacity to be monotonic:
- $\pi(\text{BLACK}) = 1/3 < \pi(\text{BLACK} \cup \text{RED}) < 1$
- $0 \leq \pi(\text{RED}) \leq \pi(\text{BLACK} \cup \text{RED})$.

When the capacities were calculated on the basis of DM statements, we found 
that some subjects violated these requirements. In the Ellsberg case, for positive 
outcomes, we had eleven subjects who violated the first restriction. We therefore 
set those values for $\pi(\text{BLACK} \cup \text{RED})$ greater than 1 to be equal to 1; values 
smaller than 1/3 were adjusted to 1/3. In the Ellsberg case, for negative 
outcomes, we had four people with $\pi(\text{RED})$ less than 0. Those values were 
replaced by 0. For the Dai Ichi Kangyo Bank case, the main focus of our paper, 
we found only few violations. For positive outcomes we calculated one value for 
$\pi(\text{RED})$ greater than 1, which we replaced by 1. For negative outcomes, we got four 
values less than 0, which we replaced by 0 \textsuperscript{13}.

In Table 4 we present the average capacities for the Ellsberg case as well as for 
the Dai Ichi Kangyo Bank case \textsuperscript{14}. To test hypothesis 1 we tested against $H_0: \pi(\text{..})$

\textsuperscript{12} According to SEU, a preference I over J would mean, that the probability of the price of the Dai 
Ichii Kangyo Bank on the 25 November 1991, being greater than 2600 Yen is more than .5. Consequently one should prefer lottery L over K, since the probability of winning in lottery K is less than .5. 

\textsuperscript{13} An alternative procedure to handle the violations of monotonicity would be to exclude these 
subjects from the analysis. However, this would induce a bias, as only subjects who are extremely 
ambiguity-averse or ambiguity-seeking would be excluded.

\textsuperscript{14} All numbers were rounded to the second digit. The tests, however, were done using four digits.
equal to probability of insufficient reason, in all 12 different cases. A* (or **) indicates a significant difference on the .05 (or .01) level. When the capacities were derived from lotteries based on positive amounts, the average capacities were significantly lower than the probabilities. The capacities reflect a considerable ambiguity-aversion, especially in the Dai Ichi Kangyo Bank case. For capacities derived from negative amounts a significant difference was only found in one case.

**Hypothesis 2.** Hypothesis 2 stated that in the Dai Ichi Kangyo Bank case capacities estimated on amount to win (DM statement) and on probability to win (ball statement) are identical. To test this hypothesis, for each person we calculated the differences for corresponding capacities and tested for their significance (H_0: \( \pi(\cdot | \text{DM statement}) - \pi(\cdot | \text{ball statement}) = 0 \)). As can be seen in Table 4, for each of the four average capacities the capacity based on changing the number of balls is larger than the capacity based on changing the amount to be won. This difference is significant for the following:

- \( \pi(\cdot >) \), positive amounts (\( \alpha < .05, t = -2.24 \))
- \( \pi(\cdot \leq) \), positive amounts (\( \alpha < .01, t = -3.10 \))
- \( \pi(\cdot >) \), negative amounts (\( \alpha < .05, t = -2.40 \)).

The above shows three out of the five cases where at least one of the two assessment procedures yielded a significant difference between capacities and probabilities (see Table 4). Remember that the largest ambiguity effect was observed for Dai Ichi Kangyo Bank with positive lotteries. For these cases we find that changing the number of balls will yield in significantly higher capacities than changing amounts of money; i.e., the degree of ambiguity aversion is larger for changing amounts than for changing the number of balls.

### Table 4
Average capacities

<table>
<thead>
<tr>
<th></th>
<th>positive outcomes</th>
<th>negative outcomes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \pi(\text{RED}) )</td>
<td>( \pi(\text{BLACK} \cup \text{RED}) )</td>
</tr>
<tr>
<td>Ellsberg</td>
<td></td>
<td></td>
</tr>
<tr>
<td>prob</td>
<td>0.33</td>
<td>0.33</td>
</tr>
<tr>
<td>DM</td>
<td>0.30 **</td>
<td>0.34</td>
</tr>
<tr>
<td>( t )</td>
<td>-2.65</td>
<td>-5.20</td>
</tr>
<tr>
<td>Dai Ichi Kangyo Bank</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \pi(\cdot &gt;) )</td>
<td>0.5</td>
<td>0.41 **</td>
</tr>
<tr>
<td>prob</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DM</td>
<td>0.41 **</td>
<td>0.40 **</td>
</tr>
<tr>
<td>( t )</td>
<td>-4.28</td>
<td>-5.15</td>
</tr>
<tr>
<td>balls</td>
<td>0.43 **</td>
<td>0.44 **</td>
</tr>
<tr>
<td>( t )</td>
<td>-5.07</td>
<td>-5.39</td>
</tr>
</tbody>
</table>
Hypothesis 3. Hypothesis 3 stated that capacities are identical for gains and losses. As with Hypothesis 2 we calculated the individual differences for corresponding capacities and tested for their significance ($H_0: \pi(.|\text{positive outcomes}) - \pi(.|\text{negative outcomes}) = 0$). Again, as we can see in Table 4, the average capacities for positive amounts are always smaller than the average capacities for negative amounts. This is significant for the following:

- $\pi(\text{RED}), \text{DM} \ (\alpha < .05, t = -2.19)$
- $\pi(\text{BLACK or RED}), \text{DM} \ (\alpha < .01, t = -3.10)$
- $\pi(>)$, DM ($\alpha < .1, t = -1.80$)
- $\pi(\leq)$, DM ($\alpha < .01, t = -3.90$)
- $\pi(>)$, ball ($\alpha < .01, t = -3.27$)
- $\pi(\leq)$, ball ($\alpha < .01, t = -4.90$)

Because the difference between capacities based on positive and negative amounts is always significantly different from zero, we must reject Hypothesis 3. Similar to decision making under risk (Kahneman and Tversky 1979), we find that people are less ambiguity-averse when we come into the domain of loss. On average we find ambiguity neutrality for losses with quite a few people being ambiguity-seeking.

4.3. Prediction

We elicited the certainty equivalent for each lottery in the validation sample (see Table 2). The goal of this part of the study (Hypothesis 4) was to investigate which theory or assessment procedure is best able to predict the certainty equivalents. As there were 69 subjects we had 1,104 observations with which to test the predictive power of CEU and the different assessment procedures.

Table 5
Average values for the validation sample

<table>
<thead>
<tr>
<th>Ellsberg</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lottery</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>EV in DM</td>
<td>33</td>
<td>33</td>
<td>-33</td>
<td>-33</td>
<td>67</td>
<td>67</td>
<td>-67</td>
<td>-67</td>
</tr>
<tr>
<td>CEU-DM</td>
<td>0.33</td>
<td>0.30</td>
<td>-0.33</td>
<td>-0.36</td>
<td>0.56</td>
<td>0.67</td>
<td>-0.66</td>
<td>-0.67</td>
</tr>
<tr>
<td>EU</td>
<td>0.33</td>
<td>0.33</td>
<td>-0.33</td>
<td>-0.33</td>
<td>0.67</td>
<td>0.67</td>
<td>-0.67</td>
<td>-0.67</td>
</tr>
<tr>
<td>U(CE)</td>
<td>0.40</td>
<td>0.36</td>
<td>-0.41</td>
<td>-0.42</td>
<td>0.65</td>
<td>0.66</td>
<td>-0.64</td>
<td>-0.64</td>
</tr>
<tr>
<td>CE in DM</td>
<td>37</td>
<td>33</td>
<td>-30</td>
<td>-31</td>
<td>62</td>
<td>62</td>
<td>-53</td>
<td>-54</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dai Ichi Kangyo Bank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lottery</td>
</tr>
<tr>
<td>EV in DM</td>
</tr>
<tr>
<td>CEU-DM</td>
</tr>
<tr>
<td>CEU-ball</td>
</tr>
<tr>
<td>EU</td>
</tr>
<tr>
<td>U(CE)</td>
</tr>
<tr>
<td>CE in DM</td>
</tr>
</tbody>
</table>
Table 5 lists the average values we elicited or calculated for each of the 16 lotteries which form the basis of testing Hypothesis 5. We have the following notation:

- **CEU-DM**: Choquet expected utility of a lottery based on capacities derived by DM statements;
- **CEU-ball**: Choquet expected utility of a lottery based on capacities derived by ball statements;
- **CE**: Certainty equivalent of a lottery;
- **U(CE)**: Utility of the certainty equivalent;
- **EU**: Expected utility of a lottery based on probabilities according to the principle of insufficient reason;
- **EV**: Expected value of a lottery based on probabilities according to the principle of insufficient reason.

When calculating the individual values of CEU-DM and CEU-ball we used the capacities that we determined for positive (or negative) outcomes if the lottery of the validation sample consisted of positive (or negative) outcomes. For each utility calculation we used the individual utility function assessed in the questionnaire; e.g., U(CE) is the (individual) utility of the stated (individual) certainty equivalent. CE was observed, however: we had to transform it to U(CE) in order to measure the observed values on the same scale as the calculated values. When determining the utility function, the average certainty equivalent for a 50–50 lottery yielding DM 0 and DM 100 was DM 46.1, thus showing some risk aversion. For negative amounts the average certainty equivalent was DM −39.8, thus showing somewhat stronger risk seeking.

To test Hypothesis 4 we take for each calculated value (CEU-DM and EU) the difference between the calculated value and the observed value transformed by u: 

- **CEU-DM - U(CE)**,
- **EU - U(CE)**.

In the case of an absolutely perfect fitted theory, the corresponding difference should be 0 for all observations. The less significantly it diverges from 0 the better the predictive power of the theory.

For **CEU-DM - U(CE)** we found that the mean is significantly smaller than zero ($\alpha < .01$, $r = -3.44$). For **EU - U(CE)** we found that the mean is significantly larger than zero ($\alpha < .05$, $t = 2.55$). Overall, neither EU nor CEU seem to be good predictors.

Contrary to EU, CEU is able to predict people's behavior when they have a non neutral attitude towards ambiguity. To sharpen our analysis we will investigate the predictive power for those 34 subjects who showed ambiguity aversion in the Ellsberg case (see Table 3). Contrary to what one would expect, EU became the best predictor; i.e., the mean of **EU - U(CE)** is not significantly different from zero, whereas **CEU-DM - U(CE)** is significantly smaller than zero ($\alpha < .01$, $t = -5.02$).

To differentiate our analysis we will consider capacities derived from positive
Table 6  
Predictive power for positive and negative Dai Ichi Kangyo Bank lotteries

<table>
<thead>
<tr>
<th></th>
<th>CEU-DM – U(CE)</th>
<th>CEU-ball – U(CE)</th>
<th>EU – U(CE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positive outcomes</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean, $\sigma^2$</td>
<td>-0.070, 0.021</td>
<td>-0.047, 0.014</td>
<td>0.002, 0.014</td>
</tr>
<tr>
<td>$t$</td>
<td>-7.93 **</td>
<td>-6.56 **</td>
<td>0.34</td>
</tr>
<tr>
<td>Negative outcomes</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean, $\sigma^2$</td>
<td>0.021, 0.038</td>
<td>0.036, 0.026</td>
<td>0.037, 0.022</td>
</tr>
<tr>
<td>$t$</td>
<td>1.74</td>
<td>3.72 **</td>
<td>4.15 **</td>
</tr>
</tbody>
</table>

and negative amounts separately. Recall that capacities derived from negative (or positive) amounts show hardly any ambiguity aversion. Since we found the strongest ambiguity aversion for Dai Ichi we will concentrate on these events.

Table 6 shows the surprising result that CEU-DM predicts best for negative amounts while EU predicts best for positive amounts. This is clearly not satisfying. While determining the capacities we have observed a strong ambiguity effect for ambiguous lotteries with positive amounts. Those effects cannot be modeled by EU.

Comparing certainty equivalents for positive and negative lotteries for Dai Ichi Kangyo Bank in Table 5 we find that the values for corresponding lotteries are remarkably similar. However, as the degree of risk seeking is on average stronger than the degree of risk aversion, and we also observed ambiguity aversion in the latter case, we can not make a general conclusion.

To gain some insight into what occurred we focus on the values for positive lotteries (Lotteries number 9, 10, 13, and 14) in Table 5. Taking risk aversion and ambiguity aversion into account, subjects gave certainty equivalents that were clearly too high. For the last lottery subjects were on average even willing to pay more than the expected value.

Let us consider, for example, Lottery 14 of the validation sample, which is identical to Lottery K of the elicitation sample. When choosing between the ambiguous Lottery K and Lottery L, a lottery with a 50-50 chance for winning DM 50 or nothing, people preferred Lottery L. To be indifferent between both lotteries, the winning amount of Lottery L (DM 50) would have to be reduced by an average of DM 9.22 (the ambiguity premium). Thus the participants were indifferent between an ambiguous lottery giving them an uncertain chance of winning DM 50 and a lottery where they could win approximately DM 41 with a .5 chance. Taking the participants’ risk aversion into account, subjects should pay less than DM 20 for Lottery K (equal to Lottery 14). Table 5 shows that subjects on average gave a certainty equivalent of DM 23.12.

A possible explanation for this might be an order effect: All participants answered the questions of the validation sample at the end. To control for an order effect we ran a second study.
5. Additional investigation

As the Dai Ichi Kangyo Bank events with positive amounts of winning showed the strongest ambiguity aversion we only considered these events in our second study. We reversed the order of the questionnaire:
- First, we asked for certainty equivalents for Lotteries 9, 10, 13, and 14.
- Next, we elicited the utility function for the interval [DM 0, DM 100].
- Finally, we let them choose between Lotteries I and J and between the Lotteries K and L.

The questionnaire was identical to the first one; however, we changed the date of the stock price to January 13th, 1992, and the amount to Yen 2000. These slightly changed lotteries are referred to as Lotteries 9', 10', 13', and 14'.

The participants were not paid to fill in the questionnaire. Thirty-five students studying mostly business and economics participated in the experiment; five participants had to be taken out of the analysis, mainly because they violated dominance requirements.

The second experiment showed similar results when we tested Hypothesis 1 and 2. Out of 30 subjects, 11 had decision patterns compatible with SEU, 15 were ambiguity averse, 3 ambiguity seeking and 1 was categorized as 'others' (see Table 3 for the results of the first experiment). The capacities based on ball statements and on DM statements were significantly smaller than .5 (see Table 4 for the first experiment):

For DM:
\[ \pi(>) = .42 \quad (\alpha < .05, t = -2.09) \]
\[ \pi(\leq) = .39 \quad (\alpha < .01, t = -3.45) \]

For balls:
\[ \pi(>) = .43 \quad (\alpha < .05, t = -2.55) \]
\[ \pi(\leq) = .41 \quad (\alpha < .01, t = -3.21) \]

For \( \pi(>) \) we found no significant difference between ball and DM capacities, whereas for \( \pi(\leq) \) the capacities derived from balls were significantly larger than those derived from DM statements (\( \alpha < .05, t = -2.10 \)).

Table 7 gives the calculated and observed values for Lotteries 9', 10', 13', and 14'. Again we tested for (i) CEU-DM - U(CE), for (ii) CEU-ball - U(CE), and for (iii) EU - U(CE) whether the mean difference was significantly different from zero. We found that (i) and (ii) were significantly less than zero. As in the first experiment the mean of EU - U(CE) was not significantly different from zero (see Table 8).

\footnote{Seventeen participants were given the lotteries of the holdout sample in the sequence 9', 10', 13', and 14', and the other 13 participants were given these lotteries in the sequence 13', 14', 9', and 10'. We found no significant differences in the certainty equivalents of these two different groups and will therefore abandon this distinction.}
Both experiments show that the certainty equivalent of an ambiguous lottery cannot be predicted with Choquet expected utility. Using only attitude towards risk (and neglecting attitude towards ambiguity) or using only attitude towards ambiguity (and neglecting attitude towards risk) we were able to predict the certainty equivalents.

As in the first experiment we will consider Lottery 14' in more detail. Our subjects on average gave a certainty equivalent of DM 22.80. In order to be indifferent between Lottery K' (equal to Lottery 14') and Lottery L the winning amount of Lottery L has to be reduced by on average DM 9.27 (ambiguity premium). Thus the participants were indifferent between the ambiguous Lottery K' and a lottery with a .5 chance of winning DM 41, a lottery with an expected value of approximately DM 20. As the participants were on average risk averse they should have a certainty equivalent clearly less than DM 20 for Lottery K'.

6. Discussion

In our study we empirically tested some behavioral implications of CEU theory. In addition we tested ways to assess capacities for ambiguous events. We described two assessment procedures to elicit and calculate capacities: changing the amount to win or changing the chances to win to make a risky lottery indifferent to the ambiguous lottery. Both assessment procedures gave results that

Table 7
Additional investigation: Average values for the validation sample

<table>
<thead>
<tr>
<th>Lottery</th>
<th>9</th>
<th>10</th>
<th>13</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>EV in DM</td>
<td>50</td>
<td>50</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>CEU-DM</td>
<td>0.42</td>
<td>0.39</td>
<td>0.24</td>
<td>0.22</td>
</tr>
<tr>
<td>CEU-ball</td>
<td>0.43</td>
<td>0.41</td>
<td>0.25</td>
<td>0.24</td>
</tr>
<tr>
<td>EU</td>
<td>0.5</td>
<td>0.5</td>
<td>0.28</td>
<td>0.28</td>
</tr>
<tr>
<td>U(CE)</td>
<td>0.50</td>
<td>0.46</td>
<td>0.29</td>
<td>0.28</td>
</tr>
<tr>
<td>CE in DM</td>
<td>45</td>
<td>41</td>
<td>24</td>
<td>23</td>
</tr>
</tbody>
</table>

Table 8
Additional investigation: Test of significance

<table>
<thead>
<tr>
<th>CEU-DM – U(CE)</th>
<th>CEU-ball – U(CE)</th>
<th>EU – U(CE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean, $\sigma^2$</td>
<td>-0.0618 *, 0.047</td>
<td>-0.0476 *, 0.043</td>
</tr>
<tr>
<td>t</td>
<td>-3.09</td>
<td>-2.50</td>
</tr>
</tbody>
</table>
were significantly different from probabilities that could be derived by the principle of insufficient reason. The degree of subadditivity can serve as a measure for the ambiguity aversion of our subjects.

We found a significant difference in attitude towards ambiguity in the gain and in the loss domain. Subjects showed a considerable aversion to ambiguity for lotteries on positive amounts and (on average) no ambiguity aversion for losses. It is therefore necessary for a preference theory to allow for different capacities in the gain and in the loss domain. Tversky and Kahneman (1992) (see also Wakker and Tversky 1993) recently proposed a theory that can model ambiguity, allows for different decision weights in the gain and loss domain, and has most of the features of prospect theory.

Our study addresses the questions of difference of capacities in the gain and loss domains. However, in testing the predictive power of CEU models, we did not find a superiority of CEU models over EU. This could be attributed to a general robustness of EU (see Currim and Sarin 1990) or to some other factors we are not aware of. This is clearly not satisfying; especially for those types of lotteries where subjects showed a strong ambiguity effect we were not able to predict with CEU better than with EU. For lotteries with positive outcomes the certainty equivalents elicited were clearly too large compared to the values calculated using capacities and utility function 16.

Using decision weights (Tversky and Kahneman 1992) instead of capacities we cannot explain the behavioral pattern shown in our study. One possible explanation is that capacities were calculated from statements where subjects had only to think about their ambiguity aversion. When giving certainty equivalents they had to both think about risk aversion and think about ambiguity aversion. Perhaps this additional complexity was enough to influence the results. Clearly, this question is worth being studied further.

Our results seem to be robust: we replicated them in a second study. Nevertheless, we want to be careful in interpreting them. Ellsberg urns are clearly not real world problems that decision makers have to consider. In comparing CEU with EU we used probabilities derived from the principle of insufficient reason. For real world decision situations it would be interesting to consider events with different degrees of perceived ambiguity and elicit probabilities for situations without ambiguity. It would also be interesting to derive other testable, qualitative predictions of CEU for individual decision making and for different economic

---

16 The effect can not be attributed to the well known preference reversal phenomenon (Lichtenstein and Slovic 1971, Grether and Plott 1979). All our lotteries were lotteries with about equal chances of winning and loosing. For these lotteries subjects on average prefer the risky lottery over the ambiguous one. In addition they state the average buying and selling prices for risky lotteries to be higher than those prices for ambiguous lotteries (Eisenberger and Weber 1993).

17 A decision weight function also allows one to transform probabilities in a risky setting. The decision weight of a .5 chance to win something can, for example, be less than .5.
settings. Considering those decision situations and testing the predictive power of CEU would be a logical step in understanding decision making under ambiguity.

References