On the relationship between Keynes’s conception of evidential weight and the Ellsberg paradox

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Abstract

A number of scholars have noted that Ellsberg’s seminal 1961 *QJE* critique of the subjective expected utility model bears certain resemblances to ideas expressed in J. M. Keynes’s 1921 *A Treatise on Probability*. Ellsberg did not mention Keynes’s work in his article and referred instead to F. Knight’s distinction between ‘risk’ and ‘uncertainty’, thus inspiring a literature on various aspects of ‘Knightian uncertainty’. Nevertheless, the recent publication of Ellsberg’s PhD dissertation [Ellsberg, D. (2001). *Risk, ambiguity and decision*. New York: Garland Publishing], submitted to the University of Harvard in 1962, reveals that Ellsberg was actually aware of Keynes’s work. This gives rise to a number of interesting questions concerning the relation between the two authors’ works. The present paper, drawing in part on a conversation with Ellsberg, attempts to answer these questions. It turns out that the ‘mystery’ of why Ellsberg did not mention Keynes in his QJE article has a simple solution, namely that his dissertation was only completed after he had written the QJE article and that he had only come across Keynes after writing the QJE article. However, it is argued that although Ellsberg recognised the link between his notion of ambiguity and Keynes’s conception of the weight of argument in his PhD dissertation, he did not fully appreciate the fact that Keynes was more concerned with ‘practical’ rather than ‘conventionalised’ choice situations. To this extent, therefore, it is fair to say that ‘Knightian uncertainty’ is in many ways closer to the ideas expressed by Keynes than by Knight, and Keynes’s actual contribution to modern decision theory has been underestimated.

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1. Introduction

The last 20 years have seen a revival of interest in the theory of probability advanced by John Maynard Keynes in his 1921 *A Treatise on Probability* (henceforth TP, CW VIII).\(^1\) This has been almost exclusively due to economists interested in the relation between Keynes’s early work on probability and the role of probability and uncertainty in his later economic writings.\(^2\) Although the leading probability theorist B. De Finetti described the TP as one of the two ‘most interesting and original works on probability’ along with Harold Jeffreys’s *Scientific Inference* (De Finetti, 1938), Keynes’s contribution to the development of the theory of probability has in fact been ‘seriously underestimated outside philosophical circles’ (Levy, 1979, p. 15) or even completely denied. For example, Jeffreys himself remarked that Keynes’s book is not ‘very successful on the constructive side, since an unwillingness to generalize the axioms has prevented Keynes from obtaining many important results’ (Jeffreys, 1961, p. 25). And L. R. Klein even went so far as to suggest that the progress of the probability theory has been ‘in no way influenced by Keynes’s work in that field…’ (Klein, 1951, p. 446, emphasis added).

Perhaps the most significant factor in the neglect of Keynes’s ideas has been the influence of the subjective conception of probability stemming from the work of Ramsey (1926), De Finetti (1937) and Savage (1954). Indeed, it is fair to say that the Subjective Expected Utility (SEU) model proposed by Savage in his 1954 *The Foundations of Statistics*, which incorporates the subjective theory of probability, has been the leading theory of choice under uncertainty in decision theory, economics and psychology over the last 50 years.

Nevertheless, over the last 20 years, an increasing number of decision theorists have questioned the prescriptions of the SEU model, both on the basis of empirical violations and philosophical arguments. Some of these people have referred to a number of ideas advanced by Keynes in the TP, and, in particular, the notion of the weight of arguments (Anand, 1991; Curley & Yates, 1989; Einhorn & Hogarth, 1985; Fox & Tversky, 1995; Gärdenfors & Sahlin, 1988; Kelsey, 1994; Tversky, 1997).

One of the starting points of the literature questioning the SEU is the seminal paper Daniel Ellsberg published in the *Quarterly Journal of Economics* in 1961.\(^3\) Ellsberg questioned some of the fundamental assumptions of the SEU model by way of a pair of thought experiments. The main purpose of Ellsberg’s article was to point out that there are some uncertainties that are not risks and to revive Knight’s distinction between ‘risk’ and ‘uncertainty’. Since Ellsberg, the objective of most of this still growing literature, usu-

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\(^1\) ‘CW VIII’ refers to the Royal Economic Society’s *Collected Works of J. M. Keynes*, volume VIII. The notation CW followed by the respective volume will be used throughout.


\(^3\) This paper was the first statement of what became known as the ‘Ellsberg Paradox’ in decision theory, which achieved notoriety in tandem with the ‘Allais Paradox’ proposed by Allais (1953).
ally labelled ‘Knightian uncertainty’, was thus to modelling decision-making which allows for such a distinction. 4

Although Ellsberg did not mention Keynes in his article, various scholars, both economists in the Keynesian tradition as well as decision theorists, have suggested a possible theoretical link with Keynes’s ideas (Camerer & Weber, 1992; Carabelli, 2002; Curley & Yates, 1989; Dequech, 2000; Dolan & Jones, 2004; Einhorn & Hogarth, 1985; Fontana & Gerrard, 2004; Fox & Tversky, 1995; Runde, 1994a). The recent publication of Ellsberg’s PhD dissertation (2001), submitted to the University of Harvard in 1962, actually reveals that Ellsberg was aware of Keynes’s work. All of this gives rise to a number of interesting questions concerning the relation between these two authors’ works.

The textual analysis of the dissertation carried out in this paper gives us interesting insights into Ellsberg’s interpretation and use of Keynes’s work, and thus answers some of these questions. But others remain unanswered. I have been greatly privileged to be able to discuss these issues with Ellsberg himself, and part of the paper will seek to address some of these remaining questions by drawing on this discussion. 5

What follows thus is an attempt to reconsider Keynes’s contribution to modern decision theory, by clarifying the relationship between his work on probability and Ellsberg’s on ambiguity. I start by describing Keynes’s theory of probability and Ellsberg’s notion of ambiguity, and then reconsider the relationship between the two authors in the light of Ellsberg’s discussion of Keynes in his PhD dissertation and my conversation with him. It turns out that Ellsberg had formulated the fundamental ideas advanced in the QJE article before having read, and thus independently of, Keynes’s work. In his PhD dissertation, in contrast, Ellsberg recognised the importance and originality of Keynes’s work, and the link between his notion of ambiguity and Keynes’s concepts of the non-comparable probabilities and the weight of arguments. But I shall argue that he did not fully appreciate the constructive part of Keynes’s analysis and dismissed it too readily. In particular, I shall argue that Ellsberg did not appreciate the fact that Keynes was primarily interested in ‘practical’ rather than ‘conventionalised’ choice situations. Ellsberg thus did not give due weight to the nature of the problem Keynes was attempting to address, a problem yet to be solved in rational decision theory. It is thus fair to say that, whilst the seminal nature of Ellsberg’s article cannot be doubted, ‘Knightian uncertainty’ is in many ways closer to the ideas expressed by Keynes than Knight and Keynes’s actual contribution to modern decision theory has been underestimated. I conclude the paper with some brief remarks about the practical relevance of the concept of evidential weight in Keynes’s economic writings.

2. Keynes’s A Treatise on Probability

Keynes’s A Treatise on Probability is a dense book, wide-ranging in scope and full of interesting insights. For the purpose of what follows, however, I will provide the reader with only a brief account of some fundamental concepts advanced by the author. 6

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4 For a review of the literature generated by Ellsberg’s QJE paper see, for instance, Camerer and Weber (1992).
5 This discussion was held after an earlier version of this paper was presented at the 2nd Storep Annual Conference, “The theory of decision in the history of economics”, University of Siena, Siena, 3–4 June 2005, where Ellsberg acted as a discussant of my paper.
6 In particular, I will focus on the concepts that are relevant to highlight the links between Keynes’s analysis and Ellsberg’s contribution. For an extensive analysis of the TP see, for instance, Carabelli (1988) and O’Donnell (1989).
2.1. Probability

In the TP, probability is conceived as a logical relation between a proposition stating some conclusion on the one hand, and a set of evidential propositions on the other. If $H$ is the conclusion of an argument and $E$ is a set of premises, then $p = H/E$ represents the degree of rational belief that the probability relation between $H$ and $E$ justifies. If $E$ makes $H$ certain, that is, if the conclusion follows directly from the premises, then $p = 1$. If the relation between $E$ and $H$ is contradictory, then $p = 0$. In cases that fall in between these two extremes, where $E$ provides some but not conclusive grounds for believing (or disbelieving) $H$, $p$ lies somewhere between 0 and 1. This approach thus makes all probabilities conditional. That is to say, it becomes simply meaningless to speak about the probability of a hypothesis, since the probability of a proposition $H$ always depends on its relationship with an actual or hypothetical body of knowledge stated in $E$.

Although probabilities are nowadays usually regarded as bearing a definite numerical value in the interval $[0, 1]$, Keynes held that, in general, degrees of belief can be measured numerically only in two particular situations: when it is possible to apply the ‘Principle of Indifference’ and when it is possible to estimate statistical frequencies. In terms of the Principle of Indifference, if each of an exhaustive and mutually exclusive list of indivisible hypothesis $H_i$ $(i = 1, 2, \ldots, n)$ is judged to be equiprobable relative to $E$, then $p(H_i/E) = 1/n$ for each $i$ (CW VIII, chapters 4 and 15).\(^7\) On the frequency view, the probability of an event $H$ is $p$ if the relative frequency of $H$ in a large number of repeated trials performed under identical conditions tends to $p$ (CW VIII, chapter 8). Keynes holds that the necessary conditions to apply either the principle of indifference or the frequency approach are quite stringent. In many cases in fact ‘no exercise of the practical judgement is possible, by which a numerical value can actually be given to the probability…’ (CW VIII, p. 29).

Although not all probability relations yield numerical values of $p$, according to Keynes, they may sometimes be amenable to binary comparisons of the form $H_1/E_1 \geq H_2/E_2$ (the symbols $>$, $\geq$ and $=$ denote the qualitative probability relations ‘more probable than’, ‘at least as probable as’ and ‘as probable as’). Moreover, he demonstrates how, from probability comparisons already given, it may be possible to derive further probability comparisons. However, he insists that some pairs of probability relations may not even be comparable in qualitative terms: ‘So far from our being able to measure them, it is not even clear that we are always able to place them in an order of magnitude’ (CW VIII, p. 29). In some cases, it may not be possible ‘to say that the degree of our rational belief is either equal to, greater than, or less than the degree of our belief in another’ (CW VIII, p. 37).\(^8\) For instance:

Consider three sets of experiments, each directed towards establishing generalization. The first is more numerous; in the second set the irrelevant conditions have been more carefully varied; in the third case the generalization in view is wider in scope than in the others. Which of these generalizations is on such evidence the most probable? There is, surely, no answer; there is neither equality nor inequality between them. We cannot always weigh the analogy against the induction, or the scope of the generalization against the bulk of the evidence in support of it. If we have more grounds than before, comparison is possible; but if the grounds in the two cases

\(^7\) For an analysis of Keynes’s application of the Principle of Indifference, see Runde (1994b) and Gillies (2000).

\(^8\) In symbols, it is not true that for all $H_1, H_2, E_1$ and $E_2$, that either $H_1/E_1 \geq H_2/E_2$ or $H_1/E_1 \leq H_2/E_2$.
are quite different, even a comparison of more or less, let alone numerical measurement, may be impossible (CW VIII, p. 31–32).

Sometimes, it is even impossible to establish if a proposition is more probable than, less probable than, or as likely as, its negation:

Is our expectation of rain, when we start out for a walk, always more likely than not, or less likely than not, or as likely as not? I am prepared to argue that on some occasions none of these alternatives hold, and that it will be an arbitrary matter to decide for or against an umbrella. If the barometer is high, but the clouds are black, it is not always rational that one should prevail over the other in our minds, or even that we should balance them... (CW VIII, p. 32).

Keynes thus provides a complex theory of probability where a variety of situations can hold. The set of probabilities is not linearly ordered and its properties can be represented by the following diagram (CW VIII, p. 42) (see Fig. 1).

Keynes explains the diagram as follows:

O represents impossibility, I certainty, and A a numerically measurable probability intermediate between O and I; U, V, W, X, Y, Z are non-numerical probabilities, of which, however, V is less than the numerical probability A, and is also less than W, X and Y. X and Y are both greater than W, and greater than V, but are not comparable with one another, or with A. V and Z are both less than W, X, and Y, but are not comparable with one another, U is not quantitatively comparable with any of the probabilities V, W, X, Y, Z (CW VIII, p. 42).

In some cases therefore, binary comparisons of non-numerical probability may be possible (for instance, probabilities V and W). Keynes discusses in particular two kinds of comparison: comparisons between different hypotheses relative to the same evidence and comparisons between the same hypothesis relative to different evidence.

These comparisons may be schematized as follows:

Judgements of Preference $H_1/E > H_2/E$,
Judgement of Indifference $H_1/E = H_2/E$,
Judgements of Relevance $H/(E&E_1) > H/E$ or $H/E > H/(E&E_1)$,
Judgments of Irrelevance $H/(E&E_1) = H/E$.

Fig. 1. Numerical, comparable and non-comparable probabilities.
Whilst judgments of preference or indifference are quite straightforward, a discussion of the definition of irrelevance requires a consideration of the notion of the ‘weight of the arguments’.

2.2. The weight of arguments

According to Keynes, the rational decision-maker should try to acquire all the information he can before making a decision (Locke’s maxim). But not all of the available information need be relevant to determining the probability of a certain proposition $H$. The general criterion of relevance proposed by Keynes is that a new piece of evidence $E_1$ is relevant to $H$, if $H/E\&E_1 > H/E$, or $H/E\&E_1 < H/E$. This definition of relevance is of course fully consistent with the standard definition of relevance adopted by the subjective theory.9

Nevertheless, in chapter 6 of the TP, Keynes raises the possibility that a piece of evidence may be relevant and yet not affect the probability. This might occur in the case in which that evidence is decomposable, and the favourably relevant parts offset the negatively relevant parts, thereby leaving the probability unchanged (CW VIII, p. 78). A few paragraph earlier he had stated:

As the relevant evidence at our disposal increases, the magnitude of the probability of the argument may either decrease or increase, according as the new knowledge strengthens the unfavourable or the favourable evidence; but something seems to have increased in either case, – we have more substantial basis upon which to rest our conclusion. I express this by saying that an accession of new evidence increases the weight of argument (CW VIII, p. 77).

This leads Keynes to a more general definition of relevance: ‘to say that a new piece of evidence is ‘relevant’ is the same thing as to say that it increases the ‘weight’ of the argument’ (CW VIII, p. 78). Or alternatively: ‘One argument has more weight than another if it is based on a greater amount of relevant evidence… The weight to speak metaphorically, measures the sum of the favourable and unfavourable evidence, the probability measures the difference’ (CW VIII, p. 84).

Keynes thus seems to be thinking about the weight of arguments as a measure of the absolute amount of relevant knowledge expressed in the evidential premises of a probability relation. Further, he emphasises that there is no necessary relation between the probability of some proposition relative to the evidence, and the weight of argument. The acquisition of new relevant evidence may change the probability in any direction but always increases the weight of arguments.

In order to explain the relevance of the notion of evidential weight Keynes provides a pedagogical example of drawing a white ball from two different urns:

\[ \text{\ldots in the first case we know that the urn contains black and white in equal proportions; in the second case the proportion of each colour is unknown, and each ball is as likely to be black as white. It is evident that in either case the probability of drawing a white ball is 1/2, but that the weight of the argument in favour of this conclusion is greater in the first case (CW VIII, p. 82).} \]

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9 For a discussion of the notion of relevance in the SEU model, see, for instance, Gärdenfors and Sahlin (1988).
It seems clear that, even if on the basis of some kind of indifference argument the probability of drawing a white ball from the two urns may be given to be the same\(^{10}\), knowledge of the exact proportions of black and white balls in the first urn provides a more substantial basis upon which to infer the probability. Obviously, in this example, the total amount of information about the known and unknown urn is quite different. In the first urn, there is only one possible distribution of black and white balls, and the evidential weight is high. In the second urn all the distributions are equally possible, and the evidential weight is low. Nevertheless, it is possible to think about intermediate levels of weight between the minimum and the maximum point. Keynes points out that although in the logical theory of probability a proposition always depends on some evidence, there may be cases where the evidence available is very scanty. He refers to the probability of such arguments as a priori probability and states:

In this case the weight of an argument is at its lowest. Starting, therefore, with minimum weight, corresponding to a priori probability the evidential weight of an argument rises, though its probability may either rise or fall, with every accession of relevant evidence (CW VIII, p. 78).

The situation Keynes appears to have in mind can be described by the example of drawing a white ball from an urn that contains black and white balls. Suppose that we start with no information at all about the distribution of white and black balls in the urn (Keynes’s second urn). As Keynes points out (again by appeal to the principle of indifference), the probability of drawing a white ball from this urn is 1/2. Due to the paucity of the evidence available I believe Keynes would refer to this probability as an a priori probability and to the weight of arguments as being at a low level. Suppose now that we can sample 10 balls from the same urn (with replacement) and that 5 balls are white and 5 balls are black. The probability of drawing a white ball would likely still be given as 1/2, but the information about the sample has increased, even if only slightly, the evidential weight.

Although weight and confidence are not the same thing, it is reasonable to regard increases in weight as leading to increases in confidence on the extent to which the probability should be relied on as a guide to conduct (Runde, 1990). That is to say, we will be more confident that our forecast is an appropriate guide to action in the case of the probability based on the sample rather than in the case of a priori probability.

If this is the case then, the decision-maker should choose his course of action by taking into consideration both the probability judgments and the evidential weight upon which these judgments are based.

3. Ellsberg’s ‘risk, ambiguity, and the Savage axioms’

On the Bayesian view associated with the SEU model, the probability of a proposition or event represents the strength of one’s degree of belief in that proposition or event (Ramsey, 1926; De Finetti, 1937). As shown by Savage (1954), probabilities can be derived operationally with reference to choices between gambles. If two gambles have the same outcome and one is preferred to the other, this means that the decision-maker attaches

\(^{10}\) The indifference argument in fact asserts that ‘if there is no known reason for predicating of our subject one rather than another of several alternatives, then relatively to such knowledge the assertions of each of these alternatives have an equal probability’ (CW VIII, p. 45).
a greater probability of winning to the alternative chosen. Moreover, if some axioms on preferences over acts with random consequences are respected, such degrees of belief conform to the axioms of the probability calculus, that is, can be represented by a unique additive probability distribution. On this approach, Knight’s distinction between ‘risk’ and ‘uncertainty’ is rendered meaningless as ‘all the uncertainties can be reduced to risks’ (Ellsberg, 1961, p. 645, emphasis in the original). 11 The decision maker always behaves ‘as if’ he assigned numerical probabilities to the events impinging on his actions (Ellsberg, 1961, p. 643).

The key contribution of Ellsberg’s famous QJE article was to show that there are some uncertainties that are not risks, thereby rejecting Savage’s idea that degree of beliefs can always be represented by additive probability distributions. In order to illustrate his argument, Ellsberg proposes a pair of hypothetical experiments. For the purpose of this paper, I shall focus exclusively on the Ellsberg ‘two-colours’ problem. 12 Suppose that there are two urns, each one containing 100 balls. The first urn is known to contain 50 red and 50 black balls, whereas the second urn is know to contain 100 balls, each of which may be either red or black (i.e. the proportion of red/black balls is unknown). The subject is asked to choose an urn and a colour, and to draw a ball from the urn you named. He or she will win $100 if the ball drawn has the colour chosen, and nothing otherwise. How to choose?

Most people are indifferent between red or black in urn 1 since the objective probability distribution of urn 1 is such that \( p(R_1) = p(B_1) = 0.5 \). Further, most people are also indifferent between betting on red or black in urn 2, presumably on the basis of some kind of indifference reasoning in the face of the fact that they do not have any more to go on; this implies \( P(R_2) = P(B_2) = 0.5 \) on the Bayesian view. Nevertheless, when asked whether they would prefer to bet on red (or black) from urn 1 or urn 2, many people prefer \( R_1 \) over \( R_2 \). This contradicts the judgments they made with respect to each of the urn individually, as it implies that \( P(R_1) > P(R_2) \) and \( P(R_1) + P(B_1) = 1 > P(R_2) + P(B_2) \). Obviously, from the Ramsey–Savage point of view, this pattern of choices is clearly inconsistent and leads to a contravention of the probability calculus: we cannot in fact infer probabilities that respect the essential properties of probability relationships from the decision-maker’s choices (Ellsberg, 1961, p. 651).

Clearly, the decision-maker’s complete ignorance about the ratio of red and black balls in urn 1 represents an extreme case and it could be argued that the pattern of choices obtained is a consequence of the paucity of information. Ellsberg replies to this objection by saying: ‘Let us suppose that you have been allowed to draw a random sample of two balls from Urn 1, and that you have drawn one red and one black. Or a sample of four: two red and two black. Such conditions do not seem to change the observed pattern of choice appreciably... The same conflicts with the axioms appear’ (Ellsberg, 1961, p. 653).

But the main problem is still to explain why people are not acting ‘as if’ they assigned numerical or even qualitative probabilities to events, or, in other words, why there are some uncertainties that are not risks. Ellsberg replies to this question in the last paragraph

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11 Most authors refer to ‘risk’, when probabilities are available to guide choice, and to ‘uncertainty’, when information is too imprecise to be summarized by probabilities (Epstein & Wang, 1994). For a clarification of Frank Knight’s distinction between risk and uncertainty, see Runde (1998).

12 The ‘three-colour urn’ example is a brilliant elaboration of the two-colours problem, useful to provide a direct test of Savage’s Independence axiom.
of his article. He points out that there is a class of choice-situations characterised by the necessity to consider ‘a third dimension of the problem of choice: the nature of one’s information concerning the relative likelihood of events’ (Ellsberg, 1961, p. 657). He calls this dimension, the ambiguity of information, being ‘a quality depending on the amount, type, reliability and unanimity of information, and giving rise to one’s degree of “confidence” in an estimate of relative likelihoods’ (Ellsberg, 1961, p. 657), and argues that in this kind of choice-situations ‘reasonable people neither wish nor tend to conform to the Savage postulates’ (Ellsberg, 1961, p. 646) and their pattern of ‘violating’ behaviour cannot be described by using standard decision rules.13

4. Traces of Keynes

Ellsberg did not refer to Keynes in his QJE article. Yet the remarkable similarities between some of the ideas advanced by the two authors are readily apparent, and a number of scholars have in fact pointed out possible theoretical links between them. For instance, economists such as Runde, refer to Ellsberg’s paper as an attempt to analyse ‘the impact of weight-like considerations in the SEU framework’ (Runde, 1994a, p. 140). Authors interested in the psychological aspects of decision-making, such as Einhorn and Hogarth, state that Keynes has already addressed ‘the notion of ambiguity by distinguishing between probability and what he called the weight of evidence’ (Einhorn & Hogarth, 1985, p. 436). Moreover, Fox and Tversky even point out that Ellsberg’s two-colour urn problem ‘was discussed by Keynes some 40 years earlier’ (Fox & Tversky, 1995, p. 586).

The inevitable question, then, is whether Ellsberg was actually aware of and possibly influenced by Keynes’s work.

The recent (2001) publication of Ellsberg’s PhD dissertation, ‘Risk, Ambiguity and Decision’, provides new insights that help answering this question. Although this dissertation is an extremely interesting work for many reasons, I shall concentrate on two here. First, from an historical point of view, it contains a detailed and reasoned report of the development and state of art of decision theory under uncertainty as it was in the 1950s. Second, from an analytical standpoint, it is an extensive elaboration of his original contribution presented in the QJE article. Let us look at this in more detail.

In the first section of chapter one of the book, entitled ‘Ambiguity and Risk’, Ellsberg defines his concept of ambiguity. He points out that he intends to deal with ‘circumstances of decision-making in which information bearing upon these relevant facts (or states, events, propositions) is scanty, marked by gaps, obscure and vague, or on the contrary plentiful and precise but highly contradictory’ (Ellsberg, 2001, p. 1). Borrowing a term from the psychological literature, he refers to information-states of this kind as ‘ambiguous’. He follows exactly the same structure of his QJE paper and refers to the work of Knight, Shackle and some proponents of the subjectivist model such as Ramsey and Savage.

The second section, entitled ‘Vagueness, Confidence and The Weight of Arguments’, constitutes a formerly completely unpublished part of Ellsberg’s work. As suggested by

13 In the last part of his paper, Ellsberg discusses the possibility of describing this pattern of behavior by using different decision rules based upon the concept of a ‘restricted Bayes solution’ developed by Hodges and Lehmann (1952).
the heading, Ellsberg discusses both the phenomenon of ambiguity and Keynes’s concept of the weight of argument. He begins by analysing jointly the contributions of J.M. Keynes, I. J. Good and B. O. Koopman. Ellsberg’s interest in the last two authors comes as no surprise since both Good and Koopman worked on developing, a few years later, some of ideas advanced by Keynes in the TP.14

Keynes, Good and Koopman propose theories of probability that are in some respects quite similar to one another. Ellsberg carefully examines these similarities. ‘The phenomena of vagueness, imprecision or lack of confidence in certain of one’s subjective judgements’ assumes a central stage in the work of Good (Ellsberg, 2001, p. 6). Like Keynes and Koopman, Good defines ‘the theory of probability as the logic (rather than the psychology) of degrees of belief’ (Ellsberg, 2001, p. 8). And like Keynes and Good, Koopman complains about ‘the particular assumption that all degrees of belief may be represented by definite, uniquely-defined numbers’ (Ellsberg, 2001, p. 7). Moreover, for Good and Koopman, ‘Degrees of beliefs are assumed (following Keynes) to be partially ordered only, i.e. some pairs of beliefs may not be comparable’ (Ellsberg, 2001, p. 8).

Having provided an overview of the theories of probability advanced by the three authors, Ellsberg focuses directly on Keynes’s fundamental ideas on probability and their relationships with his notion of ambiguity. He refers to Keynes as having ‘introduced formally the notion of non-comparability of beliefs’ (Ellsberg, 2001, p. 9). To support this statement he introduces Keynes’s barometer example:

Is our expectation of rain, when we start out for a walk, always more likely than not, or less likely than not, or as likely as not? I am prepared to argue that on some occasions none of these alternatives hold, and that it will be an arbitrary matter to decide for or against an umbrella. If the barometer is high, but the clouds are black, it is not always rational that one should prevail over the other in our minds, or even that we should balance them... (CW VIII, p. 32).

Ellsberg goes on: ‘The feeling (a) that it is sometimes difficult or impossible to compare the strength of one’s degree of belief in two given propositions (facts, events) is often closely linked to feelings: (b) that one’s information concerning one or both of these propositions is inadequate or conflicting, “ambiguous”; and (c) that one lacks “confidence” in any one stated probability comparison or definite probability distribution’ (Ellsberg, 2001, p. 11). And he continues: ‘Keynes, in particular, introduces a notion of “the weight of arguments” (as opposed to their relative probability) which seems closely related to our notion of “ambiguity”’ (Ellsberg, 2001, p. 11, emphasis added). Ellsberg then quotes Keynes on the concept of evidential weight:

The magnitude of the probability of an argument... depends upon a balance between what may be termed the favourable and unfavourable evidence; a new piece of evidence which leaves this balance unchanged, also leaves the probability of the argument unchanged. But it seems that there may be another respect in which some kind of quantitative comparison between argument is possible. This comparison turns upon a balance, not between the favourable and unfavourable evidence, but between

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14 Good explicitly admitted that he and Koopman had been much influenced by the work of J. M. Keynes (Good, 1950). As Jeffreys, Keynes in turn “was influenced by W. E. Johnson’s lectures and conversations” (Good, 1950, p. 10).
the absolute amounts of relevant knowledge and of relevant ignorance respectively. As the relevant evidence at our disposal increases, the magnitude of the probability of the argument may either decrease or increase, according as the new knowledge strengthens the unfavourable or the favourable evidence; but something seems to have increased in either case, – we have a more substantial basis upon which to rest our conclusion. I express this by saying that an accession of new evidence increases the weight of an argument. New evidence will sometimes decrease the probability of an argument, but it will always increase its “weight” (CW VIII, p. 77).

Showing some familiarity with the TP, Ellsberg thus seems to relate Keynes’s ideas about the non-comparability of probability (CW VIII, p. 32) with the weight of arguments (CW VIII, p. 77). According to Ellsberg, the difficulty, or impossibility, of comparing some degrees of belief can come from both incompleteness of information and conflicting information. He considers the weight of arguments as being capable of dealing with the problems of both the quantity and the quality of information. He confirms this interpretation by saying: ‘he (Keynes) had related the possible difficulty of comparing the probability of ‘rain’ versus ‘no rain’ to the sort of conflict of evidence we have included in the notion of ambiguity… But low “weight” he (Keynes) relates primarily to scantiness of evidence of any sort’ (Ellsberg, 2001, p. 11).

Ellsberg’s interpretation of weight is thus very broad and close to his definition of ambiguity, a quality ‘depending on the amount, type, reliability and “unanimity” of information …’ (Ellsberg, 1961, p. 657).

Ellsberg then moves on to examine the link between confidence, evidential weight, and his notion of ambiguity. He considers confidence and weight as correlative terms: ‘differences in relative weight seems related to differences in the ‘confidence’ with which we hold different opinions…’ (Ellsberg, 2001, p. 12). Further, he suggests that the concepts of ambiguity, confidence and weight are deeply interrelated. In fact he asks: ‘how may the web of action systematically reflect the varying degrees of “vagueness”, of “ambiguity/weight”, of ‘confidence’ in our judgment?’ (Ellsberg, 2001, p. 13, emphasis added).

Ellsberg claims that this question has not been addressed satisfactorily in the literature and points out that, although authors such as Knight, Savage and Keynes agree that these matters are relevant for rational decision-making, they are ‘virtually’ silent on this question (Ellsberg, 2001, p. 13). In other words, Ellsberg seems to complain about their failure ‘to develop meaningful and useful hypotheses on appropriate decision criteria for basing action, reasonably and systematically, upon ‘vague’ and ‘unsure’ opinions’ (Ellsberg, 2001, p. 12). To confirm his analysis, Ellsberg emphasizes Keynes’s hesitation about the practical relevance of the weight of arguments in the following passages:

For in deciding on a course of action, it seems plausible to suppose that we ought to take account of the weight as well as the probability of different expectations. But it is difficult to think of any clear example of this, and I do not feel sure that the theory of ‘evidential weight’ has much practical importance (CW VIII, p. 83).

If two probabilities are equal in degree, ought we, in choosing our course of action, to prefer that one which is based on a greater body of knowledge? (CW VIII, p. 345). The question appears to me to be highly perplexing, and it is difficult say much about it. But the degree of completeness of the information upon which a probability is based does seem to be relevant, as well as the actual magnitude of the probability, in making practical decisions (CW VIII, p. 345).
If, for one alternative, the available information is necessarily small, that does not seem to be a consideration which ought to be left out of account altogether (CW VIII, p. 346).

Yet Ellsberg immediately seems to soften his criticism and supports Keynes by saying: ‘When John Maynard Keynes expresses himself with so much diffidence on a subject, it is, perhaps, excusable when later theorists shy from committing themselves upon it and make some effort to build theories on other foundations; but it seems incautious of them to try to ignore that subject entirely’ (Ellsberg, 2001, p. 14).

5. Shedding light on the relationship between Ellsberg and Keynes

As can be seen from the above, at the time Ellsberg submitted his doctoral dissertation, he was aware of Keynes’s work. This then begs the question why Ellsberg did not mention Keynes in his QJE article and referred instead to other authors such as Knight, Shackle, Hurwicz, and Hodges and Lehmann. Even if the PhD dissertation was only submitted in 1962 whilst the article had been published in 1961, one could still in fact surmise that the former was written, at least in part, before or perhaps concurrently with the latter. If correct, that would mean that Ellsberg was aware of Keynes’s work when he wrote the article.

Ellsberg dispelled this conjecture in conversation with me (Siena, 04/06/2005) in the simplest manner possible: that it was only after he had submitted what became the 1961 QJE paper for publication that he read Keynes’s work. Ellsberg explained that he wanted to provide support for his paper in order to turn it into a dissertation and so resorted to Keynes’s work which he came to know through Ian Good.

This then answers the above question. It also allows us to conclude that Ellsberg was not influenced by Keynes when writing the QJE article and arrived at the ideas expressed therein independently.

But further questions, indeed ones of more interest, arise once we assess Ellsberg’s assessment of Keynes in his PhD dissertation. Ellsberg clearly recognised the importance and originality of Keynes’s work in identifying and conceptualising the problem at hand. In particular, he recognised that there was a link between his notion of ambiguity and Keynes’s concepts of the non-comparable probabilities and the weight of arguments. Against this, Ellsberg appears to have thought that even if Keynes recognized the importance of the question of ‘ambiguity/weight’ for rational decision-making, his hesitancy about the relevance of the concept of evidential weight prevented him from developing systematic decision-criteria applicable to circumstances characterised by highly incomplete information and conflict of evidence.

The main issue that arises here is whether Ellsberg was justified in dismissing the constructive part of Keynes’s analysis. In particular, since Ellsberg developed his analysis by referring to urn-type decision situations, the question is whether he fully appreciated Keynes’s contribution in respect to this kind of choice situation.

To answer this question, we now turn to the nature of Keynes’s hesitancy and failure to develop the constructive part of his analysis. I hope to show that, contrary to what Ellsberg thought, (1) Keynes’s hesitancy was not directed at the urn-type decision situations that were the subject of Ellsberg’s study and (2) Keynes actually did develop decision-criteria that can be applied to choice situations of this kind. In other words, I contend that
Ellsberg appears to have been too hasty to dismiss Keynes’s actual contribution. I shall proceed by considering points (1) and (2) in turn.

5.1. Keynes’s hesitancy

The first thing to note is that Keynes’s hesitancy is impossible to deny and must be taken seriously by anyone engaged in the study of his work. But while Ellsberg highlights the doubts expressed by Keynes in the TP, he does not enquire into their nature. This is a pity since such an enquiry leads, I shall argue, to an understanding of the type of decisional problem Keynes was dealing with.

In chapter VI of his TP, after having admitted his general hesitancy about the practical significance of the concept of evidential weight, Keynes asks if ‘we ought to make the weight of our arguments as great as possible by getting all the information we can’ (CW VIII, p. 83). And he continues: ‘It is difficult to see, however, to what point the strengthening of an argument’s weight by increasing evidence ought to be pushed’ (CW VIII, p. 83). There is in fact ‘no evident principle by which to determine how far we ought to carry our maxim of strengthening the weight of our argument. A little reflection will probably convince the reader that this is a very confusing problem’ (CW VIII, p. 83–84).

Keynes thus seems to consider the absence of a rational principle that determines when to stop the process of acquiring information as a possible objection against the use of the weight of argument.

This problem, which is the source of Keynes’s perplexities and which we could term the stopping problem, does however not apply to the urn-type decision situations analysed by Ellsberg. Let us see why this is the case.

Consider Ellsberg’s two-colour urn example again and, in particular, the urn with an unknown distribution. As already pointed out, in this case, the only information available to the decision-maker is that there are 100 balls and only two possible colours. Let us now start the process of acquiring information by picking up a sample from the urn without replacement. Since the weight of argument may here be interpreted as an expression of the amount of the information provided by the sample, the “stopping problem” consists in finding a rational principle to determine the optimal dimension of the sample. And if so, the problem can be easily solved by referring to sample theory. The problem can be analysed as follows: consider an urn containing N balls each of which may be either red or black and suppose that we want to infer something about the proportion p of red balls in the urn by considering the proportion \( f_n \) of redballs in a random sample drawn from the urn. What is the dimension n of the sample such that \( |f_n - p| \leq \delta \), where \( \delta \) is the maximum error admitted, given a certain fixed probability? Without going into the statistics, asymptotic sample theory tells us that the dimension n of the sample can be established by the following equation:

\[
 n = \frac{z_{\alpha/2}^2 Npq}{\delta^2 (N - 1) + z_{\alpha/2}^2 pq}
\]

where \( N \) is the size of the population, \( z \) is the value of the standard normal distribution, depending on the level \( \alpha \) of confidence chosen, \( \delta \) is the maximum error admitted, \( p \) is the proportion we want to estimate and \( q \) is \( 1 - p \).

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15 For an analysis of the problem, see, for instance, Cochran (1977).
The equation above thus provides a simple solution to the “stopping problem”: it is in fact possible to determine when it is rational to stop the process of increasing the dimension of the sample, that is, of strengthening the weight of an argument.\textsuperscript{16}

The perplexities that Keynes expresses thus simply do not apply to the urn-type choice situations analysed by Ellsberg. They do not apply to all choice situations where the weight of argument can be identified as a measure of the sample size and it is possible to apply standard statistical criteria to solve the “stopping problem”. Borrowing a term from Keynes, who referred to the two-colour urn example as a “conventionalised example” (CW VIII, p. 82), we might refer to decision situations of this kind as \textit{conventionalised choice situations}.

Keynes did not discuss the relevance (irrelevance) of the “stopping problem” in respect to the urn-type decision situations analysed by Ellsberg. This is not surprising since he was primarily interested in proving the effectiveness of the theory of evidential weight outside conventionalised choice situations of this type. In chapter VI of the TP, after having provided the two-colour urn example to illustrate the relevance of the concept of evidential weight, and whilst explaining the nature of his perplexities, Keynes in fact argues:

\begin{quote}
For in deciding on a course of action, it seems plausible to suppose that we have to take into account of the weight as well as the probability of different expectations, \textit{But it is difficult to think of any clear example of this, and I do not feel sure that the theory of ‘evidential weight’ has much practical importance} (CW VIII, p. 83, emphasis added).
\end{quote}

In other words: (a) Keynes was looking for an example, where the evidential weight is relevant for rational decision-making, different from conventionalised choice situations involving random drawing from urns; (b) his difficulty in finding such an example made him hesitant about the practical importance of the theory of evidential weight; and (c) this difficulty was essentially due to the “stopping problem”.

Keynes’s perplexities about the relevance of the concept of evidential weight were thus not directed to urn-type choice situations. Unfortunately, in the TP, Keynes did not clarify which kind of choice situations he had in mind. Nevertheless, it seems fair to say that he was here referring to real decision-making situations that do not involve artificial drawings from urns and where the “stopping problem” can undermine the practical effectiveness of the theory of evidential weight. We might refer to decision situations of this kind as \textit{practical choice situations}.\textsuperscript{17}

Summing up, an inquiry into the nature of Keynes’s perplexities shows that he was hesitant about the relevance of the concept of evidential weight because of the “stopping problem”.

\textsuperscript{16} Obviously, the dimension of the sample depends, given a certain level of confidence and admitted error, on the values of $p$ and $q$, and the dimension $N$ of the population. For instance, if the decision maker wanted to apply a precautionary principle, he should assume that $p$ and $q$ are equal to 1/2, so that the dimension of the sample would be, ceteris paribus, at his maximum.

\textsuperscript{17} Although Keynes did not use the word ‘practical’ consistently throughout the text, it seems suited to represent the kind of decisional problem he was dealing with. In a sister paper (Feduzi, 2004), I identify the differences between ‘practical’ and ‘conventionalized’ choice situations, and analyze their implications for the relevance of the theory of evidential weight. In particular, I argue that, since the weight of argument entails both issues of quality and quantity of information (Runde, 1990; Fioretti, 2001), the conditions under which it is possible to use the concept of evidential weight as a measure of the sample size, and thus to be able to use statistical criteria to solve the “stopping problem”, are quite stringent in practical decision-making.
problem”. Contrary to what Ellsberg thought, Keynes’s perplexities did not prevent him from developing systematic decision-criteria applicable to urn-type decision situations since his real concern was not with conventionalised choice situations but with practical decisions. And as we know now, Keynes should have not been hesitant about the effectiveness of the theory of evidential weight in respect to urn-examples in any event, since the “stopping problem” simply does not apply to decision situations of this kind.

5.2. *Keynes’s decision criterion*

Given that Keynes’s hesitancy was not in respect of conventionalised choice situations involving random drawings from urns, we can move on to consider Ellsberg’s assertion to the effect that Keynes did not develop his analysis with regards to this type of situation.

First of all, recall that Ellsberg’s critique of the SEU model is built on an example that is virtually identical to Keynes’s. It is thus surprising that although he refers to Keynes’s TP in his PhD dissertation, Ellsberg fails to mention Keynes’s two-colour urn example. Not only does Keynes provide a two-colour example that highlights the same paradox coming from a standard definition of probability pointed out by Ellsberg, but after noting that the significance of weight had been discussed in a previous chapter, he also hints at a possible rule to systematically discriminate between the two urns: “...if two probabilities are equal in degree, ought we, in choosing our course of action, to prefer that one which is based on a greater body of knowledge?” (*CW VIII*, p. 345). Despite its apparent simplicity, this proposal is not that far removed from other modern decision rules based on second order uncertainty measures such as Gärdenfors and Sahlin’s notion of ‘epistemic reliability’.18

Yet, Keynes did propose the following conventional coefficient *c* of weight and risk, that is, a general rule to combine both coefficient of risk and weight, and the probability:

\[
c = \frac{2pw}{(1 + q)(1 + w)}
\]

where *p* and *q* assume the conventional meanings and *w* measures the evidential weight (*CW VIII*, p. 348). It is not within the scope of this paper to discuss the validity of Keynes’s coefficient of weight and risk and whether it can be applied to Ellsberg/Keynes’s urn example.19 It suffices to note that Keynes’s attempt to provide a rational decision rule speaks against his alleged ‘virtual silence’ on this issue. Ellsberg’s conclusion that Keynes failed to provide a decision rule must therefore be rejected. This conclusion is accentuated when we recall that Ellsberg put Keynes’s contribution on the same plane as that of Savage and Knight.

As Ellsberg pointed out in his dissertation, even though Savage recognized that ‘there seem to be probability relations about which we feel relatively “sure” as compared with others’ (*Savage, 1954*, p. 57), he expressed his reservation by saying: ‘Some people see the vagueness phenomenon as an objection; I see it as a truth, sometimes unpleasant but not curable by a new theory’ (*Savage, 1961*, as quoted by Ellsberg, emphasis added).

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18 Gärdenfors and Sahlin state explicitly that: ‘the intuitions behind Keynes’s ’weight of evidence’ and our ‘epistemic reliability’ are related...’ (*Gärdenfors & Sahlin, 1988*, p. 321).
19 See Brady (1993) on Keynes’s conventional coefficient of risk and weight.
Knight’s attitude is somewhat similar. In *Risk, Uncertainty and Profit*, he recognized the problem by saying ‘The “degree” of certainty or of confidence felt in the conclusion after it is reached cannot be ignored, for it is of the greatest practical significance. The action which follows upon an opinion depends as much upon the amount of confidence in that opinion as it does upon the favourableness of the opinion itself’ (Knight, 1921, p. 226). He even provides the following example:

Take the case of balls in an urn. One man knows that there are red and black balls, but it is ignorant of the numbers of each; another knows that the numbers are three of the former to one of the latter. It may be argued that “to the first man” the probability of drawing a red ball is fifty-fifty, while to the second it is seventy-five to twenty-five. Or it may be contended that the probability is ‘really’ in the latter ratio, but that the first man simply does not know it (Knight, 1921, p. 219).

Notwithstanding the fact that Knight’s general insight is related in spirit to Ellsberg/Keynes’s ideas, his example is slightly different and clearly needs to be further developed to criticize the standard theory of probability. Moreover, Knight’s conclusions do not move in the direction of further criticisms: ‘It must be admitted that practically, if any decision as to conduct is involved, such as a wager, the first man would have to act on the supposition that the chance are equal’ (Knight, 1921, p. 219, emphasis added). Ellsberg in fact expressly noted that the results of his experiments ‘directly contradict Knight’s own intuition about the situation’ (Eellsberg, 2001, p. 653, emphasis added).

To this extent, it is thus paradoxical that some of the literature inspired by Ellsberg’s paper is usually labelled ‘Knightian uncertainty’; had Ellsberg been aware of Keynes’s work when he wrote the QJE paper one could in fact surmise that it would surely now be labelled ‘Keynesian uncertainty’.

Daniel Ellsberg has very kindly offered some counter-arguments to the ones I have made to the effect that he was too hasty in dismissing Keynes’s contribution.

Whilst accepting that Keynes’s perplexities concerned choice situations where the “stopping problem” applies and that he never addressed the problem himself, Ellsberg maintains that Keynes’s constructive part of his analysis is still deficient. In the first place, Ellsberg questioned the extent to which one could state that Keynes’s example is identical to his. He pointed out that Keynes did not clarify how the concept of evidential weight should affect the decision-maker’s choices and thus the axioms. Moreover, Ellsberg draws attention to the fact that in chapter VI of the TP, where Keynes provided the urn example, there is no mention of any decision criteria.

In fact, Ellsberg notes that: (a) the first decision criterion appears over two hundred pages later, and perhaps more significantly, is only hinted at and not posited affirmatively, does not explicitly refer to the urn example and can only be applied when the degrees of probability are equal; and (b) the coefficient of weight and risk is again provided, albeit rather hurriedly, over two hundred pages later and does not specifically refer to the urn example.

Ellsberg’s counter-arguments are persuasive. Keynes certainly has a very particular style of writing, and the TP, as its readers will know, is notoriously unclear. The many interpretations, sometimes contrasting, of the text are testament to this. Moreover, the

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20 See, for instance, *Runde (1990)* on the three possible definitions of evidential weight.
Ellsberg, also, could not at the time of his writing benefit from the exegetical work carried out on the TP in the last 20 years, which makes our reading of the text considerably easier. All this, compounded by the fact that Ellsberg had already formulated his fundamental ideas in the QJE article and approached Keynes only with the intent of finding support for them, led Ellsberg to his reading of the TP. Nevertheless, none of this disturbs the conclusion that the constructive part of Keynes’s analysis represents a significant contribution.

6. Some brief remarks about the practical relevance of the concept of evidential weight in Keynes’s economic writings

Ellsberg himself now recognizes that the “stopping problem” is an important issue. Since this problem sheds light on the decision problem Keynes was dealing with, it is worth considering whether Keynes eventually overcame his doubts about the practical importance of theory of evidential weight. That is to say, did Keynes find cases besides conventionalised urn examples, where the decision-maker takes into account considerations of evidential weight in the process of rational decision-making? The most direct route towards an answer to this question is, obviously, that of seeing if Keynes gave a practical role to the concept of evidential weight in his later economic writings.

We can start by noting that Keynes, as has been widely recognized in the literature, adopted a practical/realist orientation in his economic writings. Indeed, it is fair to say that important parts of his theoretical framework come from the direct observation of and engagement with real world economic phenomena. Furthermore, scholars engaged in the study of the links between Keynes’s early work on probability and his later works on economics have argued that after the publication of the TP, Keynes became even more concerned with the practical understanding of decision-making. In particular, Gerrard has argued that Keynes ‘sought to encompass his earlier, more rationalistic and academic thought within a more psychological and practical framework’ and that ‘The General Theory exemplifies Keynes’s more practical approach’ (Gerrard, 2003, p. 242).

It is thus in the analysis of the practical behaviour of economic agents that Keynes found space to provide a role to the concept of the weight of argument. As already pointed out by some scholars (see, for instance, Runde, 2003), Keynes explicitly refers to the concept of evidential weight in discussing both the state of the entrepreneur’s long-term expectation and the concept of the liquidity preference.

In chapter 12 of the General Theory, on the ‘State of Long-Term Expectation’, Keynes writes: ‘It would be foolish, in forming our expectations, to attach great weight to matters which are very uncertain’ (CW VII, p. 148). And in a footnote to this statement, he points out that ‘By ‘very uncertain’ I do not mean the same thing as ‘very improbable’ and refers to the chapter on ‘The Weight of Arguments’ in his TP.

Keynes regards the entrepreneur’s long-term expectation of prospective yield to be based partly on existing facts which can be assumed ‘to be known more or less for certain’ and partly on ‘future events which can only be forecasted with more or less of confidence’

21 The TP is in fact littered with cross references.
22 See, for instance, Runde and Mizuhara (2003) for a collection of essays on this matter.
The state of long-term expectations, upon which our decisions are based, depends thus on two different components: the most probable forecast we can make and the confidence with which we make this forecast. The outstanding fact is the extreme precariousness of the basis of knowledge upon which our estimates of prospective yields have to be made. Our knowledge of the factors which will govern the yield of an investment some years hence is usually very slight and often negligible (CW VII, p. 149). Although he does not go into much detail, Keynes thus clearly links confidence to evidential weight.23

Analogously, in chapter 17 of the General Theory, on the ‘Essential Properties of Interest and Money’, he writes:

The liquidity-premium, it will be observed, is partly similar to the risk-premium but partly different; – the difference corresponding to the difference between the best estimates we can make of probabilities and the confidence with which we make them (CW VII, p. 240).

And in a footnote to this sentence, Keynes refers to the footnote mentioned above. Keynes is even more explicit in linking confidence to evidential weight in a letter he wrote later on to Hugh Townshend:

I am rather inclined to associate risk premium with probability strictly speaking, and liquidity premium with what in my Treatise on Probability I called ‘weight’. An essential distinction is that a risk premium is expected to be rewarded on the average by an increased return at the end of the period. A liquidity premium, on the other hand, is not even expected to be so rewarded. It is a payment, not for the expectation of increased tangible income at the end of the period, but for an increased sense of comfort and confidence during the period (CW XXIX, p. 293–294).

Keynes’s theory of asset choice, much like his theory of the entrepreneur’s long-term expectation, is thus based on two different components: the actor’s probability judgments and the evidential weight upon which these judgments are based.

To sum up, it seems fair to say that in his later writings, Keynes takes the view that evidential weight plays a role in practical choice situations. Nevertheless, the fact that he makes use of the concept for analysing practical economic situations does not say anything about how, if at all, he actually, consciously overcame his doubts about the concept. In fact, he never again refers to the problem of finding a rational principle to decide where to stop the process of acquiring information, that is how much should the weight of an argument be strengthened before making a decision. I have argued elsewhere (Feduzi, 2004) that a discussion of this problem throws into light a number of interesting questions about the nature and properties of the concept of evidential weight.

7. Conclusion

Various parallels can be drawn between Ellsberg’s QJE article and the ideas expressed in Keynes’s TP. Ellsberg does not mention Keynes’s work in his article, but does discuss it in his recently published doctoral dissertation. This gives rise to a number of interesting questions about the nature and properties of the concept of evidential weight.

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questions concerning the relationship between the contributions of the two authors, which this paper, in part drawing on an instructive conversation with Ellsberg, has sought to answer.

It emerges that the fundamental ideas formulated in Ellsberg’s 1961 QJE article were arrived at before he had read, and thus independently of, Keynes’s work. Nevertheless, in his PhD dissertation, he undoubtedly recognizes the importance and originality of Keynes’s work, and the link between his notion of ambiguity and Keynes’s concepts of the non-comparable probabilities and the weight of arguments. But I have argued that Ellsberg did not fully appreciate the constructive part of Keynes’s analysis and dismissed it too readily. In particular, Ellsberg did not appreciate the fact that Keynes was more concerned with ‘practical’ rather than ‘conventionalised’ choice situations, and thus also failed to appreciate the nature of the problem Keynes was attempting to solve. To this extent, therefore, whilst the seminal nature of Ellsberg’s article is not in question, we can conclude that ‘Knightian uncertainty’ is in many ways closer to the ideas expressed by Keynes than Knight, and that Keynes’s actual contribution to modern decision theory has not been given its full due.

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