Uncertainty Aversion and Preference for Randomisation*

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In this paper we show that while individuals with non-additive beliefs may display a strict preference for randomisation in an Anscombe–Aumann framework, they will not do so in a Savage-style decision theory. Moreover they will be indifferent to randomisation, unless they have strict preferences between two randomising devices with the same probabilities. We argue that this is related to the distinction between one- and two-stage Choquet integrals. We believe our result will have implications for determining a solution concept for games with uncertainty-averse players. *Journal of Economic Literature* Classification Number: D81.

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1. INTRODUCTION

Experimental evidence (see for instance Camerer and Weber [3]) suggests that many individuals exhibit a preference for situations in which probabilities are “better known.” This phenomenon is known as *uncertainty-aversion*. Two related theories have been proposed to model it. Firstly, Schmeidler [23] and Gilboa [10] have shown that uncertainty-aversion can be modelled by strictly non-additive subjective probabilities. The expected value of utility function with respect to a non-additive probability may be expressed as a Choquet integral, [4]. Henceforth we shall

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refer to such preferences as Choquet expected utility (CEU). Secondly Gilboa and Schmeidler [11] and Kelsey [14] have modelled uncertainty-aversion by representing individuals' beliefs as convex sets of (conventional) probability distributions. In this model, actions are evaluated by the minimal value of expected utility which they yield. The resulting preferences are known as Maxmin Expected Utility (henceforth MMEU) or multiple priors.

Dow and Werlang [5] have developed a theory of games with uncertainty where players have CEU preferences. They assumed that players would choose pure strategies rather than randomising.¹ Klibanoff [15] and Lo [16] have studied games with uncertainty where players are allowed to randomise before choosing a strategy. They argue that uncertainty-aversion will imply a strict preference for randomisation between indifferent strategies if the payoffs are not comonotonic. This casts doubt on the Dow–Werlang equilibrium since it implies that a pure strategy will not always be a best response. Another implication is that a mixed equilibrium can be strict.

In this paper we examine whether uncertainty-aversion does indeed imply a strict preference for randomisation. Klibanoff and Lo use MMEU preferences in a model based on Anscombe–Aumann [1] (henceforth AA). Thus their results are not directly comparable to those of Dow and Werlang who use CEU preferences. However a similar conclusion could have been obtained for CEU preferences in an AA framework. We show that for CEU preferences, in a Savage [22] framework with a unique outcome for each state, uncertainty-aversion does not imply a strict preference for randomisation. Indeed, we can show that the decision-maker will be indifferent to randomisation provided (s)he is indifferent between different randomisations with the same probabilities.

This is related to the distinction between one- and two-stage Choquet integrals made by Sarin and Wakker [21]. The two-stage Choquet integral is used to evaluate an action in an AA model. In the first stage expectations are taken state by state with respect to the objective (roulette) probabilities. This yields a function which assigns an expected utility to each state. An overall evaluation for the action is then obtained by taking a Choquet integral of this function with respect to the state uncertainty. The one-stage Choquet integral applies to a Savage framework, in which all uncertainty (objective or subjective) is represented by a single state space. The expected utility of an action is obtained by taking a single Choquet integral. The preference for randomisation only arises with the two-stage Choquet integral, however Sarin and Wakker argue that the one-stage integral is preferable.

¹ Some other papers have taken a similar approach; see for instance, Eichberger and Kelsey [6], [7], Hendon *et al.* [13], Lo [17], Marinacci [19], and Mukerji [20].

Ghirardato [9] investigates the possibility of proving a version of the Fubini theorem for capacities. Recall the Fubini theorem says that the integral over a product space is equal to the iterated integrals on the marginals. He shows that in general if the marginal probabilities are non-additive the Fubini theorem will only hold for a class of functions which he calls “slice comonotonic.” The Fubini theorem will not hold for all functions if either marginal fails to be additive. The fact that the AA and Savage frameworks are not in general equivalent follows from this. Recall for CEU preferences the AA model consists of a product space $R \times T$ with an additive marginal on R and a marginal on T which is a convex capacity, and expected utilities are calculated by the iterated (Choquet) integral. In contrast, in a Savage framework actions are evaluated by the integral with respect to the product measure. Thus by Ghirardato’s result the two integrals will only agree on slice comonotonic actions. He does not, however, explore the implications of this for preferences over randomisations.

We do not believe the preference for randomisation argument is very intuitive. Imagine an individual faced with an Ellsberg urn. He/she can choose between £10 if a black ball is drawn and £10 if a red ball is drawn. Suppose (s)he only values either of these options at £3 due to ambiguity. Why should flipping a coin before choosing induce him/her to assign any higher value to the bets? After the flip (s)he will still be left holding a single option before the ball is drawn. We believe that if the individual is indifferent between the two options, (s)he should also be indifferent between all randomisations over them. In the rest of the paper we express this argument in formal terms.²

Our results do not imply that Lo and Klibanoff are wrong, since they use MMEU rather than CEU preferences. In Section 3.3 we show by example that even in a Savage framework, an uncertainty-averse individual with MMEU preferences may or may not have a strict preference for randomisation. One interesting aspect of this example is that the CEU and MMEU models give different predictions. As far as we are aware, there are relatively few such examples.

Organisation of the Paper

In the next section we show that strictly non-additive probabilities may imply a strict preference for randomisation in an AA framework. In Section 3

² This example was suggested to us by James Dow. One possible interpretation is that it constitutes an inappropriate application of consequentialism, see Machine [18]. Our preferred interpretation is that it illustrates that randomisation cannot eliminate the dependence of payoffs on ambiguous events.

we show in a Savage framework that there is not necessarily a preference for randomisation if beliefs are represented by a convex capacity. Moreover if the decision-maker is indifferent between equivalent randomisations then (s)he will be indifferent to randomisations. Section 4 concludes by comparing the AA and Savage models and makes some suggestions for further research.

2. PREFERENCE FOR RANDOMISATION

In this section we investigate the strict preference for randomisation in an AA framework.

Notation 2.1. There is a finite set S of states of nature. A subset of S will be referred to as an *event*. The set of possible outcomes or consequences is denoted by X . In Anscombe–Aumann [1], an *action* (or horse-lottery) is a function $h: S \rightarrow \mathcal{A}(X)$, where $\mathcal{A}(X)$ denotes the set of all probability distributions over X . These probabilities are taken to be additive and known in advance to the decision-maker. In contrast (s)he is not assumed to know the probabilities of the states. The space of actions is denoted by $H(S)$.

DEFINITION 2.1. A *capacity or non-additive probability* on S is a real-valued function ν on the subsets of S which satisfies the following properties:

$$\text{a. } A \subseteq B \Rightarrow \nu(A) \leq \nu(B), \quad \text{b. } \nu(\emptyset) = 0, \nu(S) = 1.$$

DEFINITION 2.2. A capacity is said to be *convex* if for all $A, B \subseteq S$, $\nu(A \cup B) \geq \nu(A) + \nu(B) - \nu(A \cap B)$.

DEFINITION 2.3. An event $A \subseteq S$ is said to be *dummy* if $\nu(A \cup B) = \nu(B)$, for all $B \subseteq S$.

DEFINITION 2.4. A capacity is said to be *strictly convex* if for all $A, B \subseteq S$, such that $A \not\subseteq B$, $B \not\subseteq A$, and $A \cap B$ are not dummies, $\nu(A \cup B) > \nu(A) + \nu(B) - \nu(A \cap B)$.

Remark. Convex (resp. strictly convex) capacities represent uncertainty-aversion (resp. strict uncertainty-aversion).

Notation 2.2. If $\phi: S \rightarrow \mathbb{R}$, let $\phi_{(i)}$ be the i th highest consequence of ϕ and $E_{(i)} = \phi^{-1}\phi_{(i)}$ be the event in which the consequence $\phi_{(i)}$ occurs.

TABLE I

	s_1	s_2
a	£100	£0
b	£0	£100
c	$\frac{1}{2}\text{£}100 + \frac{1}{2}\text{£}0$	$\frac{1}{2}\text{£}100 + \frac{1}{2}\text{£}0$

DEFINITION 2.5 (Choquet integral). If $\phi: S \rightarrow \mathbb{R}$, the Choquet integral of ϕ with respect to the capacity ν is defined by, $\int \phi d\nu = \phi_{(1)}\nu(E_{(1)}) + \sum_{i=2}^n \phi_{(i)}[\nu(\bigcup_{j=1}^i E_{(j)}) - \nu(\bigcup_{j=1}^{i-1} E_{(j)})]$.

The decision-maker is assumed to have preferences on $H(S)$, which may be represented by maximising $W(a) = \int E_{a(s)} u d\nu(s)$, where $E_{a(s)}$ denotes expectation with respect to the objective probability distribution $a(s)$ and u is a cardinal utility function. Expectations are first taken with respect to the objective probabilities, then a Choquet integral is taken with respect to the decision-maker's (possibly non-additive) subjective probabilities over the states. This gives the his/her overall expected utility taking into account both kinds of uncertainty. Schmeidler [23] has axiomatised these preferences.

Preference for randomisation can be illustrated with an example. Suppose that there are two states s_1 and s_2 . Consider actions a , b , and c as described in Table I. Action a assigns probability one to £100 (resp. £0) in state s_1 (resp. s_2). Action c assigns a 50:50 lottery between £100 and 0 to each state. We may normalise utility by requiring that $u(0) = 0$. The Choquet integrals for these actions are as follows, $W(a) = u(100)\nu(s_1)$, $W(b) = \nu(100)\nu(s_2)$, and $W(c) = u(100)/2$. Suppose that the decision-maker is indifferent between actions a and b . This implies that $\nu(s_1) = \nu(s_2)$. If his/her beliefs are strictly non-additive then $1/2 > \nu(s_1) = \nu(s_2)$, hence c would be preferred to both a and b . It is generally agreed that strictly non-additive probabilities model uncertainty-averse preferences. Hence in this framework uncertainty-aversion implies a strict preference for c . Klibanoff and Lo argue that c can be interpreted as a 50:50 randomisation between a and b , hence the decision-maker has a strict preference for randomisation.

The conclusion of this example is more generally true in an AA model. If a decision-maker has beliefs represented by a strictly convex capacity and two non-comonotonic actions are indifferent, (s)he will strictly prefer a convex combination of them. See the Proposition and Remark 4 in [23]. A possible motivation for this is that mixing actions averages good and bad outcomes for any given state. If a convex combination can be interpreted as a randomisation then there will be a strict preference for randomisation.

3. PURELY SUBJECTIVE PROBABILITIES

In Savage [22] the outcomes of a randomising device must be modelled explicitly as part of the description of a state. In this section we investigate attitudes to randomization in this model.

Notation 3.1. We consider a finite set S of states of nature. The set of consequences is denoted by X . An *action* is a function from S to X . Note that in this framework an action assigns a certain consequence rather than a lottery to each state. The space of all actions is denoted by $A(S)$.

We shall assume that the decision-maker has preferences $\succsim$ over $A(S)$, which may be represented by maximising $V(a) = \int u(a(s)) dv(s)$. Such preferences have been axiomatised by Gilboa [10] and Sarin and Wakker [21].

To model randomisation, we assume that the state space is a Cartesian product $S = R \times T$. We interpret R to be the space of outcomes of a randomising device and T to represent all other relevant uncertainty. There is an additive probability distribution p on R . The decision-maker's beliefs are represented by a capacity v on S .

ASSUMPTION 3.1. *We assume that for all $A \subseteq R$, $v(A \times T) = p(A)$.*

This requires beliefs to coincide with the actual probability for events whose occurrence can be determined solely from the outcome of the randomising device. The decision-maker's beliefs may fail to satisfy this assumption. However this would imply that (s)he does not perceive the R -space to be the outcomes of a randomising device, in which case the question of whether or not (s)he has a preference for randomisation is not relevant.

3.1. An Example

First we shall consider the special case where $R = \{H, T\}$ and $T = \{t_1, t_2\}$. For concreteness assume that the R -space is the set of outcomes of a flip of a fair coin, which may come up either heads H or tails T , each with probability $1/2$. Note that the probabilities associated with the coin are non-ambiguous.³ Suppose an individual is faced with an Ellsberg-style choice. As shown in Table II action d (resp. e) pays £100 if t_1 (resp. t_2) obtains and 0 otherwise.

³ One might wish to debate why the coin is perceived to be non-ambiguous, while t is perceived to be ambiguous. It could be the decision-maker has observed a large number of tosses of the coin but has little or no previous observations of t , or it could just be a property of his/her beliefs which is not necessarily based on any objective feature of the world.

TABLE II

	t_1H	t_2H	t_1T	t_2T
d	£100	0	£100	0
e	0	£100	0	£100
f	£100	0	0	£100

The randomisation d if H and e if T is represented by action f . Intuitively it is no longer clear why an uncertainty-averse individual should have a strict preference for f over either d or e . The better outcomes for f do not coincide with an unambiguous event. Assume, as in the original Ellsberg problem [8], the individual is indifferent between d and e .

The (Choquet) expected values of the three actions are $V(d) = u(100) \nu(\{t_1H, t_1T\})$, $V(e) = u(100) \nu(\{t_2H, t_2T\})$, and $V(f) = u(100) \nu(\{t_1H, t_2T\})$ (we have adopted the normalisation $u(0) = 0$). Thus a strict preference for randomisation implies $\nu(\{t_1H, t_1T\}) = \nu(\{t_2H, t_2T\}) < \nu(\{t_1H, t_2T\})$. However, this does not seem easier to justify than the opposite inequality $\nu(\{t_1H, t_1T\}) > \nu(\{t_1H, t_2T\})$, which would imply a strict preference for *not* randomising.

3.2. A More General Model

In the above example we found that if the randomising device is explicitly modelled there is no clear case in favour of a strict preference for randomisation in the CEU model. We shall now show that the decision-maker will not have a strict preference for randomisation in the case where R and T are arbitrary finite sets.

Define $A(T) = \{a \in A(S) : \forall r, r' \in R, \forall t \in T, a(r, t) = a(r', t)\}$. Actions are in $A(T)$ if and only if their outcome does not depend upon the randomising device.

DEFINITION 3.1. Suppose $a, b \in A(T)$ and $Y \subseteq R$ define an action $a_Y b : S \rightarrow X$ by $a_Y b(r, t) = a(r, t)$ if $r \in Y$, $= b(r, t)$ if $r \notin Y$.

Remark. The action $a_Y b$ is a randomisation between a and b . It corresponds to choosing action a if the randomising device selects an element of Y and b otherwise.

DEFINITION 3.2 (Symmetry). A randomising device is called *symmetric* if all its outcomes are equally likely; for all $Y \subseteq R$, $p(Y) = |Y|/n$, where $|Y|$ denotes the number of elements in the set Y and $n = |R|$.

Remark. Symmetry will be satisfied if the space R is constructed from the outcomes of a symmetric randomising device such as an unbiased die.

ASSUMPTION 3.2 (Device independence with symmetric randomisations (DISR)). *The randomising device is symmetric and $a, b \in A(T)$, $|Y| = |Z|$ implies $a_Y b \sim a_Z b$.*

Remark. This assumption says that the individual is indifferent between different randomisations between a and b with the same probabilities. As an example, DISR implies that if the randomising device is an unbiased die, (s)he is indifferent between a if the die shows 6 and b otherwise and a if 1 and b otherwise. In the form stated here, DISR only applies when the randomising device is symmetric. Under symmetry, the probability of getting action a from the randomisation $a_Y b$ only depends on $|Y|$. Symmetry is not essential for our analysis, however in its absence it would be more difficult to identify which randomisations are equivalent.

LEMMA 3.1. *Let $A, B \subseteq T$ and let $Y \subseteq R$, let Y_1, Y_2 be a partition of Y and let $P = (Y_1 \times A) \cup (Y_2 \times B)$. If v is a convex capacity on S , which satisfies assumption 3.1, then $v(P) = v(Y_1 \times A) + v(Y_2 \times B)$.*

Proof. Define sets C and D by $C = (Y_1 \times T) \cup P$, $D = (Y_2 \times T) \cup P$. By convexity

$$v(C) \geq v(Y_1 \times T) + v(P) - v(Y_1 \times A) = p(Y_1) + v(P) - v(Y_1 \times A). \quad (1)$$

$$v(D) \geq v(Y_2 \times T) + v(P) - v(Y_2 \times B) = p(Y_2) + v(P) - v(Y_2 \times B). \quad (2)$$

and

$$p(Y) = v(Y \times T) \geq v(C) + v(D) - v(P). \quad (3)$$

We obtain $p(Y) \geq p(Y_1) + p(Y_2) + v(P) - v(Y_1 \times A) - v(Y_2 \times B)$ by substituting (1) and (2) into (3). Since p is additive, $p(Y) = p(Y_1) + p(Y_2)$. Hence $v(P) \leq v(Y_1 \times A) + v(Y_2 \times B)$. However, the opposite inequality follows directly from convexity. The result follows. ■

PROPOSITION 3.1. *If the decision-maker's beliefs can be represented by a convex capacity satisfying assumption 3.1, then for all $a, b \in A(T)$, $a_Y b \succ a$ implies $b \succ b_Y a$.*

Proof. Let $u_1 > \dots > u_K$ be the utility levels yielded by actions a and b . Define $A_k = \{t \in T: u(a(r, t)) \geq u_k\}$ and $B_k = \{t \in T: u(b(r, t)) \geq u_k\}$. By definition, $V(a) = u_K + \sum_{k=1}^{K-1} v(R \times A_k)(u_k - u_{k+1})$ and $V(b) = u_K + \sum_{k=1}^{K-1} v(R \times B_k)(u_k - u_{k+1})$. Lemma 3.1 implies that $v[(Y \times A_K) \cup (\neg Y \times B_K)] = v(Y \times A_K) + v(\neg Y \times B_K)$. Therefore, $V(a_Y b) = u_K + \sum_{k=1}^{K-1} [v(Y \times A_k) + v(\neg Y \times B_k)](u_k - u_{k+1})$ and $V(b_Y a) = u_K + \sum_{k=1}^{K-1} [v(\neg Y \times A_k) + v(Y \times B_k)](u_k - u_{k+1})$. Lemma 3.1 also implies that $v(R \times A_k) = v(Y \times A_k) + v(\neg Y \times A_k)$, hence $V(a) + V(b) = V(a_Y b) + V(b_Y a)$. The result follows.

Remark. Proposition 3.1 establishes that, unlike in the AA model, it is not possible that all randomisations between a and b are strictly preferred to both a and b . Note that this proposition does not assume DISR or symmetry.

LEMMA 3.2. *If DISR is satisfied, then for all $Y \subseteq R$ and $T' \subseteq T$, $v(Y \times T') = |Y|v(r \times T')$, where r is any element of R .*

Proof. Let $a \in A(T)$ be an act such that $a(t) = x$, if $t \in T'$; $= w$, if $t \notin T'$, where $x \succ w$. Let $b \in A(T)$ be the constant act which gives consequence w . Let $Y = \{r'\}$ and $Z = \{r''\}$. Then by DISR $a_Y b \sim a_Z b$. By evaluating the Choquet integrals we find $V(a_Y b) = u(x) v(r' \times T') + u(w)(1 - v(r' \times T'))$ and $V(a_Z b) = u(x) v(r'' \times T') + u(w)(1 - v(r'' \times T'))$. Hence $v(r' \times T') = v(r'' \times T')$. By repeated application of Lemma 3.1 we may show $v(Y \times T') = \sum_{r \in R} v(r \times T')$. The result follows.

The following result shows DISR implies that uncertainty-averse individuals never have a strict preference for randomisation.

PROPOSITION 3.2. *Assume that DISR is satisfied, if $a \sim b$, then for all $Y \subseteq R$, $a \sim a_Y b$.*

Proof. We shall use the same notation as in the proof of Proposition 3.1. By Lemma 3.2,

$$V(a) = n \left\{ u_1 v(r \times A_1) + \sum_{i=2}^n u_i [v(r \times A_i) - v(r \times A_{i-1})] \right\},$$

$$V(b) = n \left\{ u_1 v(r \times B_1) + \sum_{i=2}^n u_i [v(r \times B_i) - v(r \times B_{i-1})] \right\},$$

and

$$\begin{aligned} V(a_Y b) &= u_1 [|Y| v(r \times A_1) + n' v(r \times B_1)] \\ &\quad + \sum_{i=2}^n u_i [|Y| v(r \times A_i) - |Y| v(r \times A_{i-1}) + n' v(r \times B_i) - n' v(r \times B_{i-1})], \end{aligned}$$

where $n' = n - |Y|$. Hence $V(a_Y b) = (|Y|/n) V(a) + (1 - (|Y|/n)) V(b)$, from which the result follows.

Remark. Note that in fact we have proved a stronger result than actually stated, since we have shown that preferences are linear in the non-ambiguous probabilities generated by the randomising device. Proposition 3.2 implies

that if individuals sometimes have a strict preference for randomisation then they will not be indifferent between equivalent randomisations, in which case it would be difficult to construct a theory of games.

3.3. *The Multiple Priors Model*

In the example of Section 3.1, a strict preference for randomisation is possible if individuals have MMEU preferences. Consider the following set of additive beliefs: $\{p: p(t_1 H) = p(t_1 T) = 1/4 - \varepsilon, p(t_2 H) = p(t_2 T) = 1/4 + \varepsilon: -\alpha \leq \varepsilon \leq \alpha\}$, where $1/4 > \alpha > 0$.⁴ With these beliefs the decision-maker will evaluate actions d , e , and f as $W(d) = W(e) = u(100)(1/2 - 2\alpha)$, $W(f) = 1/2u(100)$. Hence, the randomisation strategy f is preferred to the two non-randomised strategies. Note that these preferences satisfy DISR.

They also satisfy the equivalent of assumption 3.2 for MMEU preferences since $p(H) = 1/2$ for all probability distributions in the above set of beliefs. It is easy to show that this set of beliefs is not the core of a convex capacity. This also follows from Propositions 3.1 and 3.2 above.

4. CONCLUSION

4.1. *Comparison of the Savage and Anscombe–Aumann Frameworks*

This paper has shown that individuals with strictly non-additive subjective probabilities may have a strict preference for randomisation within an AA framework but not in a Savage framework. This result does not of itself determine whether uncertainty-aversion implies a strict preference for randomisation; however, our preference is for the purely subjective approach. First, we believe that explicitly modelling the randomising device is clearly superior.

Second, AA's objective probabilities were introduced to simplify the analysis, by making it easier to scale the utility function. Their use is innocuous in expected utility, where essentially the same form of preferences obtained in either model. However as the present paper has shown, with non-additive probabilities the preferences have different properties in the two models. More generally our results suggest the desirability of reappraising the relative merits of AA and Savage frameworks for studying non-expected utility theories.

Even in an AA framework, it is still possible to include the outcomes of some randomising devices in the description of the states. Our results suggest that an uncertainty-averse decision-maker will always have a strict preference for randomisations determined by the objective probabilities. However, under our assumptions, (s)he will not have a strict preference for

⁴ We are grateful to Kin Chung Lo for suggesting this example.

randomisations, where the outcomes of the device are included in the description of the state. We believe that different treatment of two kinds of randomisation with the same probabilities is implausible and is an argument for not using the AA framework in this context. As mentioned before, a theory in which individuals were not indifferent between equivalent randomisations would be hard to apply in conventional economic models.

The analysis of the present paper has used the non-additive probability model of Schmeidler [23] and Gilboa [10]. The example in Section 3.3 shows that a strict preference for randomisation is possible with MMEU preferences, thus our results cannot be extended to that model. On the other hand in a Savage framework it is also possible to have multiple priors beliefs, which do not express a strict preference for randomisation. This follows directly from the previous sections, since CEU preferences with a convex capacity are a special case of, MMEU. Hence, even for MMEU preferences, uncertainty-aversion does not imply a strict preference for randomisation. At present there is no axiomatisation of MMEU in a Savage framework. Thus it is not clear whether this would provide a justification for assuming a strict preference for randomisation.

It has been argued that the most promising model of uncertainty-aversion is given by preferences which satisfy both the MMEU and CEU axioms. See, for instance, Gilboa and Schmeidler [12]. In a Savage framework such preferences will not display a strict preference for randomisation.

One possible objection to our results is that the Savage theory does not allow for the possibility of an objective randomising device. We believe that this is an incorrect interpretation of this theory. Savage himself certainly intended it to model *all* kinds of uncertainty (including “objective” randomising devices). We are not aware of any subsequent work, which shows that it is not capable of modelling them. A serious problem with this objection is that it assumes that the decision-maker should treat “objective” probabilities differently than non-ambiguous subjective probabilities. We see no justification for this assertion. Note that objective and subjective randomising devices with the same probabilities would be treated the same way by expected utility preferences.

4.2. *Experimental Evidence*

It is not clear that the evidence supports a strict preference for randomisation. Many Ellsberg-type experiments have been performed. See for instance Bernasconi and Loomes [2]. For a survey see Camerer and Weber [3, Section 3.1]. Nearly all have found uncertainty-averse behaviour. Most subjects would have had access to simple randomising devices, e.g., watches and coins. If they had a strict preference for randomisation, they would have used them. As a result uncertainty-averse

choices would not be observed. (The difference between SEU and CEU individuals would be that the latter would perform randomisation while the former would not.⁵ This is not in accord with the actual evidence that individuals systematically discount ambiguous alternatives.

There is scope for further experimental work in this area, since at present experimental research only provides indirect evidence about preferences for randomisation. We believe it would be worth using experiments to examine cases where the Savage and AA frameworks make different predictions. This would clarify which provides the better description of uncertainty-averse behaviour. In particular, it may be worth investigating whether subjects faced with ambiguity are prepared to pay to randomise.

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⁵ Assuming that there is a small cost to performing a randomisation.

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