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Technological Progress, Investment Frictions and Business Cycle: New Insights from a Neoclassical Growth Model

M. DONADELLI, V. MOJTAHED, A. PARADISO*

Abstract

This paper examines whether there is a direct link between investment frictions and technological progress. An augmented version of a standard stochastic Solow model is presented. In this novel version the TFP is a function of a set of "ad hoc" variables in deviation from their equilibrium trend: (i) relative price of investment goods with respect to consumption goods (i.e. investment frictions); (ii) human capital index and (iii) trade openness. Empirical results show that investment frictions have an important role in influencing productivity growth. This finding may help in solving an important puzzle raised by the recent business cycle accounting literature, which points out that frictions have a marginal role in driving business cycles. The continuous fluctuations around the long-run trend of exogenous variables entering as driving forces in the technological progress implies that productivity shocks are state dependent. In other words, the true effect on the stock of knowledge and output depends on the exogenous variables’ cyclical phase. This provides novel, realistic and country-specific policy implications.

Keywords: technological progress, macroeconomic fluctuations, investment frictions, trade openness, education

JEL Codes: E32, C32, O47.

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1. Introduction

The Solow (1956) Neoclassical Growth Model (NGM), despite its age continues to provide a basic paradigm for the theoretical and empirical literature. For example, there are a large number of papers aimed at testing the convergence implications of this model (Lee et al, 1997; Caggiano and Leonida, 2007), the determinants of growth using "augmented versions" of the standard framework (Dalggaard and Strulik, 2013), or simply to build future growth patterns (Jorgenson et al., 2013; Byrne et al., 2013; Fernald, 2014; Fernald and Jones, 2014). An important research topic linked to the NGM is the Real Business Cycle (RBC) literature, which is based on the neoclassical model of capital accumulation augmented by technology shocks. RBC research became very famous as its concepts and methods diffused largely into mainstream macroeconomics. A recent evolution of this framework (known as Business Cycle Accounting, BCA), studies the economic crisis, showing that investment frictions have a marginal role in generating fluctuations. This seems in sharp contrast to the theory of Keynes (1936), Minsky (1957) and Kindleberger (1978) and generates a puzzle into the BCA results. In this paper, introducing an augmented and innovative version of Solow model, we offer an explanation for the marginal role played by investment frictions in generating business fluctuations.

A first attempt to model macroeconomic fluctuations implementing Solow’s approach in an equilibrium model, can be found in the seminal contribution of Kydland and Prescott (1982). In their approach, the economy is modeled using a standard dynamic stochastic general equilibrium (DSGE) model. In this setting, the performance of the model is validated based on the proximity of the simulated outcome to the empirical data. A different method to study business cycle fluctuations has been recently proposed by Mulligan (2002) and Chari et al. (2007). This method, known as BCA, relies on the original RBC model and assumes that the economy is described as a standard Solow growth model with time-varying wedges of efficiency, investment, labor, government expenditure, and consumption. These wedges, which represent frictions or shocks, are calculated from the equilibrium conditions and are measured so that the outcome of the model exactly matches the actual data. In other words, the frictions are obtained so the model exactly replicates the data.\(^1\) The advantage of this approach is that one can learn about the importance of the wedges in driving business cycles (Mulligan, 2002; Chari et al, 2007).

Despite the initial debate about its validity (see, among others, Christiano and Davis, 2006; Justiniano et al., 2010), the BCA approach has become rather popular in the RBC literature. It has

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\(^1\) In a standard RBC model, differently, the frictions are pre-calculated by researchers using plausible assumptions. The model is thus simulated using pre-determined friction values. As a result, simulated moments differ from empirical ones.
been employed, for example, to investigate (i) the Great Depression and the 1982 recession in the US (Chari et al., 2007); (ii) the recession of the 1990s in Japan (Kobayashi and Inaba, 2006), and (iii) the 1980s recession in the UK (Kersting, 2008). All these BCA studies are in accordance with the following findings: (i) productivity, i.e. the efficiency wedge, has an important role in explaining most of the output fluctuations in those years; (ii) investment frictions\(^2\) have a marginal role in explaining business cycle fluctuations. In some cases, such as in Chari et al. (2007), the investment frictions move in the opposite direction with respect to the output cyclical pattern. A more recent research conducted by Chadha and Warren (2013) on the 2008-2009 crisis in the UK confirms the two above findings.

The marginal role played by investment frictions in explaining recent recession episodes seems in contrast with a large part of theoretical literature on business cycles. As discussed in Gertler (1988) and Brunnermeier et al (2012), there is a long-standing tradition in macroeconomics arguing that the status quo of the credit market may represent a key driver of macroeconomic fluctuations (see Fisher, 1933; Keynes, 1936; Minsky, 1957; Kindleberger, 1978).\(^3\)

The literature focusing mainly on the linkages between investment frictions and macroeconomic variables have identified a “puzzle” in the BCA approach. An answer may lie in the relationship between financial and investment frictions, and aggregate productivity (see Buera and Moll, 2015). One important source of investment frictions derives from frictions in the credit market, as suggested by Chari et al (2007).\(^4\) The idea here is that firms may not be able to credibly commit to repay their debts. Loosely speaking, if many firms are credit-constrained, the aggregate productivity can be significantly smaller than the one generated in an economy where firms can commit to pay back the debt. The relationship between financial/investment frictions and aggregate productivity has received increasingly attention. A list of quantitative exercises focusing on this relationship can be found in Buera and Shin (2013), Gilchrist et al. (2014), Moll (2014), Buera and Moll (2015). Relative little research, however, has empirically examined the effects of investment frictions on the TFP and thus on macroeconomic fluctuations.\(^5\)

\(^2\) Chari et al (2007) show that investment frictions are the result of credit market frictions. In particular, in the appendix they demonstrate that a model with financial frictions (see Bernanke et al., 1999) is equivalent to a model with investment frictions. Therefore, both the RBC and BCA literature use investment and financial frictions interchangeable.

\(^3\) See also Gertler (1989), and Kiyotaki and Moore (1997).

\(^4\) See also footnote 2.

\(^5\) An exception is Li (2015) who investigates empirically the importance of firms borrowing constraints on TFP dynamics using micro data for Japan. Our paper differs from this work in different ways. First, our analysis uses time series techniques for three countries (i.e., USA, UK, Japan). Second, we use the relative price of investment to consumption as a measure of investment frictions as suggested by Greenwood et al (2000), Chari et al (1997), Fuentes and Morales (2011). Third, our estimation equations are derived from a stochastic version of Solow model.
The main contribution of this paper is to empirically examine whether investment frictions affect aggregate productivity. Our analysis focuses on the US, the UK and Japan (JPN) and relies on a stochastic version of the standard Solow’s neoclassical growth model (NGM). Our main result confirms the intuition of Buera and Moll (2015), that is, financial frictions can have sizable adverse effects on TFP. In contrast to the previous literature on this topic, our paper offers a tractable analysis of a stochastic version of NGM where, in addition to technology shocks, exogenous driving forces explain the evolution of stock of knowledge. The use of this approach is motivated by two main factors. First, it is easy to implement empirically with time series techniques (see for example Fuentes and Morales, 2011; Fernald and Jones, 2014). Second, differently from the NGM, our analysis provides insights on the specific policy an authority could use to stimulate economic growth. In a standard neoclassical growth framework, policies mainly rely on increases in the saving rate and decreases in the population growth rate. In our setting, by modeling the stock of knowledge as a function of “ad hoc” exogenous variables, we are able to discuss policy implications more rigorously and realistically. This even if we compare our analysis with a standard RBC one.6

In our setting, both a technology shock and a set of “ad hoc” exogenous driving forces influence the technology path. These forces are investment frictions, human capital index, and trade openness. As suggested by the literature (see, among others, Chari et al, 1997; Greenwood et al, 2000; Fuentes and Morales, 2011), investment frictions are captured by the relative price of investment goods with respect to consumption goods (i.e. the ratio of PPI to CPI). In the spirit of Psacharopoulos (1994), the human capital index is measured by the average years of schooling corrected for the rate of return. Finally, the sum of imports and exports divided by GDP is used as proxy for trade openness. These two variables are considered by the literature as key variables in explaining growth and productivity process across economies (Bergheim, 2008). In line with the BCA literature, we assume that movements of the aforementioned exogenous variables around their long-run trend determine the oscillation patterns of technology growth and, then, output per worker growth. Our long-run trends are obtained via a simple Hodrick-Prescott (HP) filter.7

In this paper, we first discuss the equilibrium properties of the model and then estimate the growth rate of output per employee of the stochastic Solow model jointly with the newly proposed TFP

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6 Quantitative papers are able to study few factors focusing on the main mechanism under investigation (i.e., financial frictions-aggregate productivity link in our case), whereas a NGM setting is flexible enough to capture different forces driving the productivity path (see for example the formalization of TFP used by Fuentes and Morales (2011), Kumar and Pacheco (2012), Lucchetta and Paradiso (2014)). Notice also that Solow (2008) raises some doubts on the use of DSGE suggesting that small models such as the simple neoclassical growth model may be better “at understanding some little piece of the (macro-)economic mechanism”.

7 In empirical literature, methods such as HP attempts to extract equilibrium values using past and future observations of macroeconomic series (Asea and Mendoza, 1994; Berger and Kempa, 2014).
growth structure. Since the TFP is not observable, a latent variable approach is used. The employed state-space model displays very plausible and statistically significant results. In particular, they confirm that investment frictions enter as a key business cycle driving force indirectly through its effect on aggregate productivity. By ignoring this result, we may lose an important piece of information regarding the real sources of macroeconomic fluctuations. The proposed TFP structure – given by the sum of standard technology shocks and exogenous variables in deviations from their long-run equilibrium as sources of productivity growth - and the presented empirical results provide novel, realistic and country-specific insights for policymakers.

By simulating the model using previously estimated parameters, we show that the continuous oscillations of exogenous variables around their long-run equilibrium trend imply that productivity shocks are state dependent. In this respect, the effect on the stock of knowledge and output depends on the cyclical phase of the exogenous variables pattern. In other words, the same shock generates different macroeconomic outcomes.

The rest of the paper is organized as follows. Section 2 describes the model and defines the steady state. Section 3 presents the estimation results using Kalman filter techniques. Section 4 presents simulations of the model using the parameters calibration derived from the empirical part. Section 5 concludes.

2. Theoretical framework

The standard neoclassical growth theory predicts that an economy moves toward the steady-state equilibrium - where there can be permanent economic growth - only if there is technological progress. Thus, in the presence of an increase in the saving rate or a decrease in the rate of population growth the economy will move to a higher steady state level of output per capita. Both factors are exogenous and determine only temporary growth above that generated by the technological progress. Loosely speaking, higher growth rate respect to the technology progress materializes only during the transition period where the economy moves from the old to the new steady state. In this framework, of course, macroeconomic fluctuations do not show up: the output growth process generated by this model is flat and corresponds to the technology growth. Temporary deviations from this horizontal path may show up only in the presence of changes in the saving or population growth.
The RBC theory integrates growth and business cycle within a stochastic framework, where the fluctuations are generated by continuous exogenous shocks to aggregate technology. A non-exhaustive list of stochastic versions of the NGM include: Mirman (1972, 1973), Binder and Pesaran (1999), Lee et al. (1997), and Novales et al. (2008). In the spirit of these studies, we propose a framework where the economy's deterministic component is continuously disturbed by transitory deviations. Differently from the previous literature, these deviations are not exclusively generated by exogenous shocks but depend also on the oscillations of a set of relevant exogenous variables around their respective long-run trends. In particular, we assume that the evolution of the stock of knowledge is also influenced by the ratio between the variable's actual value and its long-run trend.

The logic here is rather simple and relates to the $q$-theory applied to investment decisions: (i) if the ratio is greater than one (i.e., in our case the actual current value is higher than the long-run trend), a positive impact on the TFP materializes; (ii) if the ratio is lower than one (i.e., the current value is lower than the long-run trend), we observe a negative impact on the TFP; (iii) the ratio oscillates continuously over time around the value one (i.e., the equilibrium value). A natural question arises: what are the implications of this particular setting on the dynamics of the stock of knowledge? If we define the stock of knowledge as a function of (i) a deterministic and constant trend, $\gamma$, and (ii) a set of ad hoc exogenous variables adjusted for their deviation to a long-run trend, then the growth rate of the stock of knowledge will fluctuates around a long-run equilibrium determined by the constant term $\gamma$. This fluctuation materializes even if no shocks occur and, in this way, we differ from a standard stochastic NGM setting. In particular, the macroeconomic fluctuations are not only the results of purely exogenous unidentifiable shocks, but also depend on identifiable exogenous variables, which can be eventually directly influenced by the policymaker.

Another important departure from the standard NGM relies on the formalization of the persistent term in the evolution of the stock of knowledge. Specifically, it is assumed that the exogenous shock follows an autoregressive process (see also Binder and Pesaran, 1999; Novales et al., 2008). This implies that only the shocks can influence future states through the autoregressive parameter, whereas the other growth determinants do not embody a memory component.

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8 The contribution by Kydland and Prescott (1982) starts from the idea of analyzing (within the neoclassical framework of optimizing agents) the behavior of an economy converging toward its long-run equilibrium that is continually shocked by random disturbances. Solow in an interview in 2002 (Clement, 2002) criticized the fundamental assumption of the basic RBC that implies a single coherent consumer maximizes the utility spanning over an infinite horizon.
However, an important effect is neglected in this setting. This relies on the idea that innovation—regardless from the source—may generate new opportunities for future innovation improvements. Notice also that Pancrazi (2011) and Pancrazi and Vukotic (2013) shows that the presence of an autoregressive component in the TFP process is crucial in explaining some stylized facts of the business cycle as well as in influencing policymakers’ interventions. For these reasons, we follow Fuentes and Morales (2011) in assuming that the persistence component is embodied in the growth rate of the stock of knowledge and thus it does not affect only the shock process.

2.1. Stochastic shocks in the technology progress

The literature employing NGMs assumes that the stock of knowledge evolves according to the following rule (see, among others, Binder and Pesaran, 1999; Novales et al., 2008):

\[ A_t = (1 + \gamma)A_{t-1} \cdot \varphi_t \]  

(1)

where \( \varphi_t = \varphi_{t-1}^\rho e^{\mu_t}, \mu_t \sim N(0, \sigma_\mu^2), 0 < \rho < 1 \). Applying the natural logs we obtain:

\[ \ln A_t = \ln(1 + \gamma) + \ln A_{t-1} + \ln \varphi_t \]  

(2)

\[ \Delta \ln A_t \approx \gamma + \ln \varphi_t \]  

(3)

where \( \ln \varphi_t = \rho \ln \varphi_{t-1} + \mu_t \).

According to this branch of literature, randomness shows up through a productivity shock displaying a first-order autoregressive structure. The cyclical pattern of technology growth is simply the result of the continuous lagged-exogenous shocks. In the steady state (SS) - here defined by assuming that each stochastic process takes its mean value in every period (see Brock and Mirman, 1973; Novales et al., 2008) - it is immediate to show that \( \Delta \ln A_{SS} = \gamma \).

2.2. Exogenous driving forces in the technology progress

As aforementioned, we assume that the growth rate of the stock of knowledge consists of (i) a constant exogenous growth term, \( \gamma \); (ii) a set of ad hoc additional exogenous variables \( g_{t,t} (\forall i \in N) \); and (iii) an exogenous shock \( \mu_t \). Formally,

\[ A_t = (1 + \gamma_t)A_{t-1} \]  

(4)

with

\[ (1 + \gamma_t) = (1 + \gamma) \prod_{t=1}^{N} g_{t,t}^\tau_t (1 + \gamma_{t-1})^\rho e^{\mu_t}. \]  

(5)
The term \( g_{i,t} (\forall i \in N) \) represents a strictly positive exogenous variable capturing the effect on the stock of knowledge at time \( t \) and \( \tau_i \) is the magnitude of this effect. In our setting, \( g_{i,t} = X_{i,t}/\bar{X}_{i,t} \) (\( \forall i \in N \)) where \( \bar{X}_{i,t} \) denotes the trend value of variable \( X_i \) at time \( t \). This suggests that in the long-run each variable will take its trend values (i.e. \( X_{i,t} = \bar{X}_{i,t} \)) implying the following: \( g_{i,t} = 1 \). The parameter \( \rho \) (with \( 0 < \rho < 1 \)) captures the persistence of the time-varying growth process, and \( \mu_t \) is the technology shock with \( \mu_t \sim N(0, \sigma^2_{\mu}) \).

Using natural logs, Eq.(5) can be written as:

\[
\gamma_t \approx \gamma + \sum_{i=1}^{N} \tau_i \ln g_{i,t} + \rho \gamma_{t-1} + \mu_t
\]  

(6)

According to our hypothesis, the growth rate of the stock of knowledge growth depends on (i) a constant term, \( \gamma \); (ii) a set of \( N \) exogenous variables expressed as a log-difference from their long-term trend (i.e., \( \ln g_{i,t} = \ln X_{i,t} - \ln \bar{X}_{i,t} \)); (iii) an autoregressive term, \( \rho \); and (iv) an exogenous shock, \( \mu_t \).

What happens in the SS? The SS is obtained by assuming that each stochastic process and all variables are equal to their average values (Brock and Mirman, 1973; Novales et al., 2008). Thus, the stochastic productivity shock \( \mu_t \) is equal to its mean value of 0 and \( g_{i,t}^{\tau_i} \) takes the following SS form:

\[
\left( \frac{X_{i,t}}{\bar{X}_{i,t}} \right)^{\tau_i} = 1 \Rightarrow \tau_i \left( \ln X_{i,t} - \ln \bar{X}_{i,t} \right) = 0, \forall i \in N
\]  

(7)

Finally, by iterating backward Eq. (6):

\[
\Delta \ln A_{SS} \approx \frac{\gamma}{(1-\rho)}, \quad (\forall \ t)
\]  

(8)

Notice that Eq. (6) allows us to generalize the inclusion of each exogenous variable at different time periods. For instance, we may have some variables influencing the economy with a different lag structures, \( g_{i,t-h} \). In other words, they influence the stock of knowledge only after some periods \( h \) (with \( 1 < h < t \)). This may happen in those circumstances where some variables deviating from their respective trends take time to produce real effects on technology growth. As we will discuss later in Section 3, this is the case of education. An increase in years of schooling translates in an increase in productivity with huge lags because for younger cohorts it takes time to replace older cohorts in labor market. Assuming that \( N_2 \) (with \( N_2 = N - N_1 \)) variables enter with a lag \( t - h \). Eq. (6) can be then re-written as follows:
\[ \gamma_t \approx \gamma + \sum_{i=1}^{N_1} \tau_i \ln g_{i,t} + \sum_{j=1}^{N_2} \tau_j \ln g_{j,t-h} + \rho \gamma_{t-1} + \mu_t \]  

(9)

2.3. The model

The representative firm produces output using a Cobb-Douglas technology which satisfies the Inada conditions:

\[ Y_t = K_t^\alpha (B_t L_t)^{1-\alpha}, \alpha \in (0,1) \]  

(10)

where \( L \) and \( K \) are the labor and the stock of physical capital at the beginning of time \( t \), respectively. \( B_t L_t \) represents the effective units of labor, with \( B_t \) being the labor augmenting variable which we consider it to be proportional to the stock of knowledge \( A_t \):

\[ B_t \equiv A_t^{\frac{1}{1-\alpha}} \equiv A_t^\beta, \]  

(11)

With \( \beta \equiv 1/(1-\alpha) \). Therefore, \( \Delta \ln B_t \approx \beta \Delta \ln A_t \). The stock of knowledge \( A_t \), described in Section 2.2, can be written as:

\[ A_t = (1 + \gamma)A_{t-1}Z_t, \]  

(12)

where \( Z_t \equiv \prod_{i=1}^{N} g_{i,t}^{\tau_i} (1 + \gamma_{t-1})^\rho e^{\mu t} \). The stock of physical capital evolves according to

\[ K_{t+1} = I_t + (1 - \delta)K_t, \delta \in (0,1) \]  

(13)

where \( I_t \) is the investment and \( \delta \) is the constant depreciation rate of capital. It is assumed the economy is in a full employment regime. This implies that the labor supply is equal to population. The population grows at the exogenous rate \( n \):

\[ L_{t+1} = (1 + n)L_t \]  

(14)

We close the economy by assuming that the households save a fraction of their income and the investment and saving coincides:

\[ S_t = sY_t, \ s \in (0,1) \]  

(15)

\[ I_t = S_t \]  

(16)

For our following discussions, it is useful to introduce some additional notations. We define the capital and output per capita as

\[ k_t \equiv \frac{K_t}{L_t}, \ y_t \equiv \frac{Y_t}{L_t} \]  

(17)

Also capital and income can be expressed per effective of labor input
\[ \hat{k}_t \equiv \frac{K_t}{B_t L_t}, \quad \hat{y}_t \equiv \frac{Y_t}{B_t L_t} \quad (18) \]

Therefore, we can rewrite the production function as

\[ \hat{y}_t = \hat{k}_t^\alpha \quad (19) \]

From the capital accumulation equation we have

\[ K_{t+1} = sY_t + (1 - \delta)K_t \]

\[ \hat{k}_{t+1} = \frac{sY_t}{B_{t+1}L_{t+1}} + \frac{(1-\delta)K_t}{B_{t+1}L_{t+1}} \quad (20) \]

Using Eqs. (11), (12), (14), capital can take the following form:

\[ \hat{k}_{t+1} = \frac{sY_t}{(1+n)\lambda_t(1+\gamma)\beta \Omega^\beta Z_{t+1}} + \frac{(1-\delta)K_t}{(1+n)\lambda_t(1+\gamma)\beta \Omega^\beta Z_{t+1}} \quad (21) \]

Manipulating Eq. (21) and using Eq. (19), we get

\[ \hat{k}_{t+1} = \frac{1}{(1+n)(1+\gamma)\beta \Omega^\beta Z_{t+1}} (s\hat{k}_t^\alpha + (1 - \delta)\hat{k}_t) \quad (22) \]

### 2.3.1 Steady State calculation

In this section we define the SS properties of the proposed NGM. Notice that, we refer to a “counterfactual state” as a state that an economy will never reach or maintain. As aforementioned, our SS is defined as the state that an economy would tend, if the stochastic shock was permanently at its long run (or mean) value in every single period. To solve for the SS, we set all stochastic shocks and variables to their long-run values. Thus, \( \hat{k}_t = \hat{k}_{t+1} = k_{SS} \) and \( Z_{t+1} \) has to be equal to its average long-run value:

\[ Z_{t+1} = \frac{(1+\gamma_{t+1})}{(1+\gamma)} \Rightarrow Z_{SS} = \frac{(1+\gamma_{SS})}{(1+\gamma)} = \Omega \quad \text{(i.e. constant)} \quad (23) \]

Using Eq. (22)

\[ \hat{k}_{SS} = \frac{1}{(1+n)(1+\gamma)\beta \Omega^\beta} (s\hat{k}_{SS}^\alpha + (1 - \delta)\hat{k}_{SS}) \quad (24) \]

and dividing both sides for \( \hat{k}_{SS} \)

\[ (1 + n)(1 + \gamma)\beta \Omega^\beta = s\hat{k}_{SS}^{\alpha - 1} + (1 - \delta) \quad (25) \]

we get the following SS value for capital:
\[ k_{SS} = \left( \frac{s}{(1+n)(1+\gamma)\Omega^\beta(1-\delta)} \right) \]  

(26)

From Eq. (26) it is immediate to see that

\[ \frac{\Delta k_{SS}}{k_{SS}} = 0 \]  

(27)

Given Eq. (19), we have

\[ y_{SS} = \left( \frac{s}{(1+n)(1+\gamma)\Omega^\beta(1-\delta)} \right)^{\alpha\beta} \Rightarrow \frac{\Delta y_{SS}}{y_{SS}} = 0 \]  

(28)

Since \( k_{SS} = k_{SS,t}/B_{SS,t} \) and \( \Delta lnB_{SS} = \frac{\beta y}{(1-\rho)} \), then

\[ k_{SS,t} = B_{SS,t} \left( \frac{s}{(1+n)(1+\gamma)\Omega^\beta(1-\delta)} \right)^{\beta} \Rightarrow \frac{\Delta k_{SS,t}}{k_{SS,t}} \approx \Delta lnB_{SS} + \frac{\Delta k_{SS}}{k_{SS}} = \frac{\beta y}{(1-\rho)} \]  

(29)

Finally, the SS value of output can be written as follows:

\[ y_{SS,t} = B_{SS,t} \left( \frac{s}{(1+n)(1+\gamma)\Omega^\beta(1-\delta)} \right)^{\alpha\beta} \Rightarrow \frac{\Delta y_{SS,t}}{y_{SS,t}} \approx \Delta lnB_{SS} + \frac{\Delta y_{SS}}{y_{SS}} = \frac{\beta y}{(1-\rho)} \]  

(30)

3. Empirical model and results

The state space model is a useful tool to represent a dynamic system involving unobservable variables such as \( \gamma \). In the spirit of Fuentes and Morales (2011) and in line with the proposed theoretical setting, the system can be represented as follows:

\[
\begin{align*}
\Delta ln y_t &= \Delta ln A_t + \alpha \Delta ln k_t + \epsilon_t \\
\Delta ln A_t &= \gamma + \sum_{i=1}^{N} \tau_i ln g_{i,t} + \rho \Delta ln A_{t-1} + \mu_t
\end{align*}
\]  

(31.a)  (31.b)

with \( 0 < \rho < 1 \), \( \left( \epsilon_t, \mu_t \right) \sim NID \left( \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2_e & \sigma_{\epsilon\mu} \\ \sigma_{\epsilon\mu} & \sigma^2_\mu \end{pmatrix} \right) \).

In this system, Eq. (31.a) is the signal equation, whereas Eq. (31.b) represents the state equation; \( y \) is the observable per worker GDP, \( k \) is the observable capital/labor ratio, and \( \gamma \) is the unobservable long-run technology growth. \( ln g_{i,t} \) (\( \forall i \in N \)) represents exogenous variables in log deviations from their trend (i.e. \( ln g_{i,t} = ln X_{i,t} - ln \bar{X}_{i,t} \)). The set of employed “ad hoc” explanatory variables includes: (i) trade openness; (ii) human capital index; and (iii) relative price of equipment to
consumption goods. For a brief explanation about the linkages between these variables and the TFP, see Appendix A. Details on the construction of the $g_{i,t}$ and data sources are given in Appendix B.\(^9\)

The state space model defined in Eqs. (31.a)-(31.b) is estimated using the Kalman filter algorithm. The Kalman filter is a recursive algorithm for the linear projection of the state vector based on observed data that allows to define the likelihood function of the model based on the prediction error decomposition (Harvey, 1990). Numerical optimization routines are used to maximize the likelihood function. The initial values for parameters and the magnitude of autoregressive coefficient are important for the optimization process to converge. In other words, the Kalman filter needs a prior. As in other maximization procedures, if the chosen initial values are far away from the true values the system will not converge. There is no standard procedure to overcome this problem. If possible, a practical solution is to run first an OLS estimation that will give a broad idea about the parameter values (see Wells, 1996). Yet, this does address the issue related to the initial value for the variances $\sigma^2_\varepsilon$ and $\sigma^2_\mu$. The usual practical solution is to set extremely high values so as to go away from the initial values of the parameters very quickly. However, leaving the variances free to oscillate may produce a problem, as suggested by Stock (2001) and Stock and Watson (1999). The authors show that when the true variances of unobserved variables are small, the maximum likelihood estimates of the variances generally tend towards zero. Effectively, the estimation procedure gets trapped at a corner solution involving no fluctuations in the unobservable variable. This is why most of the literature tends to fix the value of the variance of the unobserved variable, or alternatively the signal-to-noise ratio (i.e., the ratio $\lambda = \sigma_\mu/\sigma_\varepsilon$).

Based on this premise, a two-step procedure is implemented to estimate the system (31.a)-(31.b). In the first step a univariate version of the system is estimated (i.e., a version of (31.a)-(31.b) without the component $\Sigma_{t=1}^{N} \tau_i l \alpha g_{i,t}$). Then, the estimated stock of knowledge is used to run a general-to-specific (GETS) approach for selecting the most significant variables at the appropriate lag structure (Hendry, 1995; Campos et al., 2005). The GETS estimate is also used to identify the starting values of the coefficients and fixing $\rho$ in the numerical optimization routine used to maximize the likelihood function in (31.a)-(31.b). In the second step the system (31.a)-(31.b) is estimated according to the identified exogenous variables, the starting values, and the $\rho$ obtained in the previous step. Results, based on two different signal-to-noise ratios suggested by empirical literature, are presented in Table 1.

\(^9\) Notice that our approach is rather flexible. Of course, a model with additional exogenous variables can be easily implemented. In this paper, we rely on variables meant to be important drivers of the business cycle. We leave the challenge of finding further business cycle driving variables for future research.
The results are satisfactory. All estimated parameters have the expected sign. As suggested by both the theoretical and empirical literature the capital share $\alpha$ is around 1/3 (see Sørensen and Whitta-Jacobsen, 2010, chapter 2). An exception is for JPN, which displays a relatively low $\alpha$ in the case of a higher signal-to-noise-ratio (see Table 1, Panel B). Notice that our findings are in line with those reported in Jones (2003) (see Figure 1, page 8), where Japan’s capital share (with the correction suggested by Gollin, 2002) is lower than in the other OECD countries.

Another noteworthy finding is that the human capital – measured as average years of schooling - enters with a sizable lag (more than 10-years) in the USA and the UK. This means that, for example, an increase in the average years of schooling translates in an increase in output after a decade. This may be due to the fact, for instance, that younger cohorts do not immediately replace older cohorts (Appiah and McMahon, 2002). A similar result can be found in Sylwester (2000) and Keller (2006).


<table>
<thead>
<tr>
<th></th>
<th>USA</th>
<th>UK</th>
<th>JPN</th>
<th>USA</th>
<th>UK</th>
<th>JPN</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.3733</td>
<td>0.4221</td>
<td>0.2886</td>
<td>0.4012</td>
<td>0.4159</td>
<td>0.1848</td>
</tr>
<tr>
<td></td>
<td>(0.0279)</td>
<td>(0.0357)</td>
<td>(0.0485)</td>
<td>(0.0367)</td>
<td>(0.0513)</td>
<td>(0.0830)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.0050</td>
<td>0.0042</td>
<td>0.0016</td>
<td>0.0049</td>
<td>0.0042</td>
<td>0.0022</td>
</tr>
<tr>
<td></td>
<td>(0.0002)</td>
<td>(0.0003)</td>
<td>(0.0003)</td>
<td>(0.0004)</td>
<td>(0.0004)</td>
<td>(0.0007)</td>
</tr>
<tr>
<td>$\tau_{hci}$</td>
<td>0.7346 (0.0485)</td>
<td>0.2588 (0.0873)</td>
<td>-</td>
<td>0.7505 (0.0776)</td>
<td>0.2203 (0.1321)</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>[lag = 19]</td>
<td>[lag = 18]</td>
<td></td>
<td>[lag = 19]</td>
<td>[lag = 18]</td>
<td></td>
</tr>
<tr>
<td>$\tau_{open}$</td>
<td>0.0312 (0.0053)</td>
<td>0.0152 (0.0057)</td>
<td>0.0538 (0.0054)</td>
<td>0.0378 (0.0075)</td>
<td>0.0107 (0.0078)</td>
<td>0.0516 (0.0074)</td>
</tr>
<tr>
<td></td>
<td>[lag = 3]</td>
<td>[lag = 2]</td>
<td>[lag = 3]</td>
<td>[lag = 3]</td>
<td>[lag = 2]</td>
<td>[lag = 3]</td>
</tr>
<tr>
<td>$\tau_{rel}$</td>
<td>-0.0876 (0.0126)</td>
<td>-0.1533 (0.0189)</td>
<td>-0.0277 (0.0173)</td>
<td>-0.0916 (0.0184)</td>
<td>-0.1833 (0.0279)</td>
<td>-0.0288 (0.0226)</td>
</tr>
<tr>
<td></td>
<td>[lag = 1]</td>
<td>[lag = 0]</td>
<td>[lag = 1]</td>
<td>[lag = 1]</td>
<td>[lag = 0]</td>
<td>[lag = 1]</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.57</td>
<td>0.58</td>
<td>0.81</td>
<td>0.57</td>
<td>0.58</td>
<td>0.81</td>
</tr>
</tbody>
</table>

Notes: $\lambda = \sigma_{\mu}/\sigma_{\varepsilon}$. The explanatory variables entering in $\Delta \ln A_t$ (trade openness, open, the human capital index, hci, and the relative price of equipment to consumption goods, rel) are selected according to a general-to-specific (GETS) A univariate version of (31.a)-(31.b) is estimated first to get a preliminary value for $\Delta \ln A_t$. $\Delta \ln A_t$ is then regressed against the selected explanatory variables. A GETS selecting procedure is used to (i) identify the lag-structure (optimally selected lags are reported in square brackets); (ii) fix $\rho$ and (iii) and identify the parameter initial values. A dummy in 1974 is added in all countries in Eq. (31.a). P-values are reported in parentheses.

Of course, entries in Table 1 allow us to recover the SS growth rate. The SS growth rate of stock of knowledge ($\Delta \ln A_{SS} = \beta \gamma/(1 - \rho)$) for USA is 0.019, whereas for UK is 0.017 in both cases of high and low $\lambda$. For JPN, the SS growth rate of stock of knowledge varies from 0.012 (low signal-to-noise ratio) to 0.014 (high signal-to-noise-ratio).
4. Simulations

The properties of the stock of knowledge pattern (as defined in Section 2.2) will influence also the main macroeconomic aggregates (i.e. output, consumption and investment). The simulation of the stock of knowledge progress is carried out as follows. First, in the spirit of King and Rebelo (1993), we simulate the quantities defined in Eq. (4) and Eq. (6) by assuming the following exogenous variables’ cyclical pattern form:\(^1\)

\[\ln X_{i,t} - \ln X_{i,t-1} \approx d_i \cdot \cos(\omega_i t - \theta_i)\]  

(32)

where \(t = \text{time}, \frac{\theta}{\omega} = \text{phase displacement}, d = \text{amplitude of the cycle}, \frac{2\pi}{\omega} = \text{period of the cycle.} \)

Appendix B shows that this approximation works well for cyclical patterns obtained via HP filter. Second, three simulations are run for examining different aspects of the technology process: (i) the regular cycles generated by sinusoids processes over a long-time period; (ii) the pattern over the last 40 years generated by the three components of stock of knowledge (i.e., deterministic, regular cycles, shocks); (iii) the same pattern as in (ii) but expressed in growth rates. The specific parameters choices rely on the estimated values reported in Table 2.

Table 2: The stock of knowledge simulation. Parameters calibration.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>USA</th>
<th>UK</th>
<th>JPN</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\gamma)</td>
<td>0.005</td>
<td>0.004</td>
<td>0.002</td>
</tr>
<tr>
<td>(\rho)</td>
<td>0.570</td>
<td>0.580</td>
<td>0.810</td>
</tr>
<tr>
<td>(\sigma_{\mu}^2)</td>
<td>0.0005</td>
<td>0.0005</td>
<td>0.0002</td>
</tr>
<tr>
<td>(A_0)</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>(\tau_{open})</td>
<td>0.031</td>
<td>0.015</td>
<td>0.054</td>
</tr>
<tr>
<td>(d_{open})</td>
<td>0.041</td>
<td>0.039</td>
<td>0.077</td>
</tr>
<tr>
<td>(\omega_{open})</td>
<td>0.393</td>
<td>0.571</td>
<td>0.785</td>
</tr>
<tr>
<td>(\theta_{open})</td>
<td>4.250</td>
<td>4.050</td>
<td>5.850</td>
</tr>
<tr>
<td>(\tau_{ PCI})</td>
<td>0.735</td>
<td>0.259</td>
<td>-</td>
</tr>
<tr>
<td>(d_{ PCI})</td>
<td>0.003</td>
<td>0.001</td>
<td>-</td>
</tr>
<tr>
<td>(\omega_{ PCI})</td>
<td>0.419</td>
<td>0.449</td>
<td>-</td>
</tr>
<tr>
<td>(\theta_{ PCI})</td>
<td>5.900</td>
<td>2.800</td>
<td>-</td>
</tr>
<tr>
<td>(\tau_{ rel})</td>
<td>-0.088</td>
<td>-0.153</td>
<td>-0.028</td>
</tr>
<tr>
<td>(d_{ rel})</td>
<td>0.016</td>
<td>0.010</td>
<td>0.020</td>
</tr>
<tr>
<td>(\omega_{ rel})</td>
<td>0.393</td>
<td>0.571</td>
<td>0.698</td>
</tr>
<tr>
<td>(\theta_{ rel})</td>
<td>2.910</td>
<td>3.484</td>
<td>2.800</td>
</tr>
</tbody>
</table>

Notes: The parameter \(\theta\) for all countries, is adjusted respect to Appendix B estimates to embody the lags of the explanatory variable entering in the state equation (see Table 1).

In Figure 1, we plot the regular cyclical term and all the three components of stock of knowledge (expressed in levels and growth rates) for USA, UK, JPN. The cycle term (formed by the sum of the three sinusoids functions approximating the HP cycle of the three exogenous variables) is periodic.

\(^1\) Cyclical components are obtained via a standard HP filter.
for all countries. However, the periodicity of fluctuations differs among countries.\textsuperscript{11} For example, in the USA we have an oscillating pattern with repeated increasing and decreasing fluctuations. A similar but weaker oscillating pattern is exhibited by JPN. A more regular cyclical structure is shown by UK. In Figure 1 we also plot the simulated pattern of stock of knowledge in levels and growth rates, over the period 1970-2011. We observe that in the USA and JPN the cyclical part oscillates continuously around the deterministic term. For the UK this effect is less evident.

**Figure 1.** Stock of knowledge pattern simulation

\textbf{USA: Regular cycle over the long-run}

\textbf{USA: Components of stock of knowledge (level)}

\textsuperscript{11} This result is due to the fact that different sinusoids functions - with different amplitude, phase displacement and periodicity - form the cyclical component.
USA: Components of stock of knowledge (growth)

UK: Regular cycle over the long-run

UK: Components of stock of knowledge (level)
UK: Components of stock of knowledge (growth)

JPN: Regular cycle over the long-run
Figure 2 depicts the effect of a 1% negative impulse shock on the growth rate of the stock of knowledge. Given the presence of cyclical term, the effect is state dependent. That is, the same shock may have a different effect because of the different amplitude and periodicity of the cyclical component. In other words, the macroeconomic outcome of a 1% shock at time $t$ is different from the one at time $t + j$ (with $j > 1$).

Production function frameworks are usually used for discussing about productivity and output growth policy interventions among governments (see for example Carone et al., 2006). Policymakers should be aware of this important mechanism in particular when they implement policies aimed at influencing productivity and output growth. Our analysis points out that the extent of these interventions is determined by the cyclical phase of the variables influencing the technology progress. Hence, forecasts based upon the productivity pattern, which do not consider this state dependence is biased. The size of this bias is increasing in the peaks and troughs of the cycle.
Figure 2. The effects of technology shocks on the growth rate of the stock of knowledge
UK: Negative shock occurred in the peak phase

JPN: Negative shock occurred in the trough phase

JPN: Negative shock occurred in the peak phase
5. Concluding remarks

This study examines whether investment frictions affect the technological progress in a new framework. To do so, we introduce a stochastic version of the Solow's neoclassical model where the growth rate of the stock of knowledge is a function of a set of "ad hoc" exogenous variables defined by their deviations from their long-run trends. The selected variables are: (i) PPI divided by CPI (i.e. a measure of investment friction); (ii) human capital index and (iii) trade openness. Our empirical results show that the marginal role played by investment frictions in driving business cycles, observed in previous BCA studies (Chari et al, 2007; Kobayashi and Inaba, 2006; Kersting, 2008; Chadha and Warren, 2013), may be caused by the direct relationship between investment frictions and TFP. This relates also to the seminal contribution in the macroeconomic literature (Keynes, 1936; Minsky, 1957; Kindleberger, 1978).

Overall, the approach presented in this paper contributes to the existing literature in three main directions. First, differently from the classical Solow model, it allows identifying a set of specific macro variables influencing the technological progress. Second, it provides novel, realistic and country-specific policy implications; an aspect neglected in standard NGM and DSGE frameworks. Third, it shows that the continuous oscillations of exogenous driving forces around the respective long-run equilibrium generate state-dependent productivity shocks. In other words, the transition path toward steady state will depend on the cyclical phase of the exogenous variables pattern.
References


Appendix A: Data sources and variable descriptions

Data on real GDP, employment, real stock of capital, are taken from Penn World Table (Heston, Summers, and Aten 2002). GDP and stock of capital are expressed in 2005 US$. In our estimation results, per worker capital stock growth as ensued from PWT is lagged by one period. This is because the capital stock constructed by Heston et al. (2002) uses the perpetual inventory model, assuming that investment enters without lags in the law of motion of capital:

\[ K_{t+1} = (1 - \delta)K_t + I_{t+1} \]

Our hypothesis (more in line with the conventional view that investments take time to transform in capital (see for example (Sørensen & Whitta-Jacobsen, 2010, chapter 3) assume instead that investment enter with a lag in the formation of capital stock (see Eq. (13))). Assuming that investments enter with a lag in the law of motion of capital stock it is similar to assume that capital stock growth, as calculated using PWT data, is lagged by one period. We reconstruct the capital stock growth for USA assuming a constant depreciation rate \( \delta \); this series is plotted in Fig. A1 against the capital stock growth taken from PWT. The plot shows that the two series have a close timing and pattern, proving that our reconstruction works well. If we build, assuming the same depreciation rate \( \delta \), a stock of capital growth according with Eq. (13) (i.e., if we assume that investment enters with a lag in the law of motion of capital stock) and plot this series with the PWT version lagged by one period, we see that the two variables have the same dynamic and timing (Fig. A2). This demonstrated that lagging the PWT stock of capital growth is almost identical to assume (especially from 1970) that investments enter with a time lag in the law of motion of capital stock. A similar argument holds for the UK and JPN, but for brevity’s sake we only focus on the US.

Figure A.1: The evolution of the growth rate of the stock of capital

Notes: \( K^* \) is the capital stock reconstructed assuming a depreciation rate equal to 3% and according to this rule: \( K_{t+1} = (1 - \delta_i)K_t + I_{t+1} \). \( K^{**} \) is the capital stock reconstructed according to this rule: \( K_{t+1} = (1 - \delta_i)K_t + I_t \).

Human capital index is constructed transforming the data on average years of schooling \( s \) (Barro and Lee 2013) in rate of returns using the correction suggested by (Psacharopoulos 1994):

\[ hci_{i,t} = e^{\theta(s_{i,t})} \]

where \( hci_{i,t} \) is the index of human capital per worker for the country \( i \). \( \theta \) is a function, with a zero intercept and a slope of 0.134 through the 4th year of education, 0.101 for the next 4 years, and 0.068 for the education beyond the 8th year. Since data on average years of schooling are available...
in 5-year intervals, the series have to be interpolated. We use a spline function that provides better interpolation accuracy than the linear case (i.e., the interpolation error is smaller).

Trade openness is calculated using PWT database. The openness is the sum of nominal import and nominal export divided by nominal GDP.

The relative price of investment goods with respect to the consumption goods is defined as the ratio of PPI to CPI. PPI and CPI are taken from OECD Statistics Data.

Trade openness, human capital index, and relative price are defined as deviation from their trend, and then expressed in natural logs. The Hodrick-Prescott (HP) filter (with a smoothing parameter is equal to 100) is then used to extract variables’ trend. It is well known that HP suffers from an end-point bias. The usual way (Kaiser and Maravall, 2001) to deal this problem is to extend the series with ARIMA forecasts until the 2014 for avoiding the problem.

**Appendix B: Identifying a regular cycle component for macro variables**

In general, it is possible to approximate a natural log of variable in difference respect to its trend as a function of cosine:

\[ \ln X_t - \ln \bar{X}_t \approx d \cdot \cos(\omega t - \theta) + \epsilon_t \]

where \( t = \) time, \( \theta \) = phase displacement, \( d \) = amplitude of the cycle, \( \frac{2\pi}{\omega} \) = period of the cycle (N).

Having obtained \( \ln X_t - \ln \bar{X}_t \) via HP filter, a simple regression shows that with appropriate values of \( \omega \) and \( \theta \) the cosine function explains well the cyclical fluctuations of a macro variable. In particular, we run regressions (i.e., the parameter \( d \) is estimated by OLS) for a wide range of values of \( \omega \) and \( \theta \) and the regression with higher R-square is selected as the most appropriate approximation of equation above. Since the frequency range for discrete-time sinusoids is finite with duration of \( 2\pi \), \( \theta \in [0,2\pi] \). Discrete-time sinusoids require that the period \( N \) is necessarily an integer; this implies that \( \omega \in \left[ \frac{2\pi}{1}, \frac{2\pi}{2}, \cdots \frac{2\pi}{20} \right] \), with the assumption that the potential maximum period of a cycle is 20 years (i.e., two complete cycles in 40 years).

Results for the US, the UK, and JPN are reported in Table C1 for the macro variables entering as potential explanatory variables in stock of knowledge equation (i.e., equation 5 in the text).
Table C.1: Sinusoidal approximations of HP filtered macro variables.

<table>
<thead>
<tr>
<th>Country</th>
<th>Variable 1</th>
<th>Parameter 1</th>
<th>Parameter 2</th>
<th>Parameter 3</th>
<th>N</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>USA</td>
<td>ln(hci) − ln(hci)</td>
<td>0.003 (0.000)</td>
<td>0.419</td>
<td>4.223</td>
<td>15</td>
<td>0.554</td>
</tr>
<tr>
<td></td>
<td>ln(open) − ln(open)</td>
<td>0.041 (0.011)</td>
<td>0.393</td>
<td>3.133</td>
<td>16</td>
<td>0.230</td>
</tr>
<tr>
<td></td>
<td>ln(rel) − ln(rel)</td>
<td>0.016 (0.003)</td>
<td>0.393</td>
<td>3.373</td>
<td>16</td>
<td>0.405</td>
</tr>
<tr>
<td>UK</td>
<td>ln(hci) − ln(hci)</td>
<td>0.001 (0.000)</td>
<td>0.449</td>
<td>0.940</td>
<td>14</td>
<td>0.355</td>
</tr>
<tr>
<td></td>
<td>ln(open) − ln(open)</td>
<td>0.038 (0.009)</td>
<td>0.571</td>
<td>3.032</td>
<td>11</td>
<td>0.272</td>
</tr>
<tr>
<td></td>
<td>ln(rel) − ln(rel)</td>
<td>0.010 (0.002)</td>
<td>0.571</td>
<td>3.484</td>
<td>11</td>
<td>0.321</td>
</tr>
<tr>
<td>JPN</td>
<td>ln(hci) − ln(hci)</td>
<td>0.002 (0.000)</td>
<td>0.571</td>
<td>2.103</td>
<td>11</td>
<td>0.343</td>
</tr>
<tr>
<td></td>
<td>ln(open) − ln(open)</td>
<td>0.077 (0.018)</td>
<td>0.785</td>
<td>3.635</td>
<td>8</td>
<td>0.296</td>
</tr>
<tr>
<td></td>
<td>ln(rel) − ln(rel)</td>
<td>0.020 (0.005)</td>
<td>0.698</td>
<td>2.000</td>
<td>9</td>
<td>0.273</td>
</tr>
</tbody>
</table>