CeLEG Working Paper Series

ONE EMU FISCAL POLICY FOR THE EURO

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Working Paper No. 02/16
April 2016

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One EMU Fiscal Policy for the EURO*

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LUISS Guido Carli

April 10, 2016

Abstract

We build a Two-Country Open-Economy New-Keynesian DSGE model of a Currency Union to study the effects of fiscal policy coordination, by evaluating the stabilization properties of different degrees of fiscal policy coordination, in a setting where the union-wide monetary policy affects fiscal policies and viceversa, because of price rigidities and distortionary taxation. We calibrate the model to represent two groups of countries in the European Economic and Monetary Union and run numerical simulations of the model under a range of alternative shocks and under alternative scenarios for fiscal policy. We also compare welfare under the different scenarios, bringing to policy conclusions for the proper macroeconomic management of a Currency Union. We find that: a) coordinating fiscal policy by targeting net exports, rather than output, produces more stable dynamics, b) consolidating government budget constraints across countries and moving tax rates jointly provides greater stabilization, c) taxes on wage income are exponentially more distortionary than taxes on firm sales. Our policy prescriptions for the Eurozone are then to use fiscal policy to reduce international demand imbalances, either by stabilizing trade flows across countries or by creating some form of fiscal union or both, while avoiding the excessive use of labour taxes, in favour of sales taxes.

JEL classification: E62, E63, F42, F45, E12

Keywords: Fiscal Policy, International Policy Coordination, Monetary Union, New Keynesian

*A special thanks to Pierpaolo Benigno for useful research guidance and to Giovanna Vallanti for helpful academic guidance. We would like to thank Konstantinos Tsinikos and seminar participants in LUISS Guido Carli for useful comments and suggestions. All errors are our own. We acknowledge financial support through the research project FIRSTRUN (Grant Agreement 649261) funded by the Horizon 2020 Framework Programme of the European Union.

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Contents

1	Introduction	1
2	A Two-Country Currency Union Model	3
2.1	Households	4
2.2	International Identities and Assumptions	11
2.3	International Risk-Sharing	13
2.4	Firms	14
2.5	Net Exports, Net Foreign Assets and the Balance of Payments	18
2.6	Central Bank and Monetary Policy	20
2.7	Government and Fiscal Policy in a Pure Currency Union	21
2.8	Government and Fiscal Policy in a Coordinated Currency Union	25
2.9	Government and Fiscal Policy in a Full Fiscal Union	28
3	Calibration	30
3.1	Sensitivity Analysis	34
4	Welfare Analysis and Optimal Fiscal Policy Parameters	35
4.1	Welfare Gains based on Consumption Equivalent Variations	37
4.2	Welfare Gains based on an ad hoc Loss Function	39
5	Numerical Simulations	41
5.1	Fiscal Policy Instruments	41
5.2	Financing Fiscal Policy	44
5.3	Fiscal Policy Coordination	45
6	Fiscal Shocks and Fiscal Multipliers and Spillovers	48
6.1	Steady State Fiscal Multipliers and Spillovers	48
6.2	Short-Run Fiscal Multipliers and Spillovers	49
6.3	Fiscal Shocks and Fiscal Policy Coordination	51
7	The Case for International Goods as Complements	54
7.1	Fiscal Policy Coordination and Welfare: a Comparison	54
7.2	Fiscal Shocks and Fiscal Multipliers and Spillovers	56
8	Conclusions and Possible Extensions	58

A Mathematical Appendix	64
A.1 Equilibrium Conditions	64
A.2 The Steady State	70

1 Introduction

Are there gains from fiscal policy coordination in a monetary union subject to alternative shocks? Does this create a scope for a centralized fiscal capacity in the EMU?

Given a single monetary policy in the European Economic and Monetary Union (EMU), country-specific shocks cannot be addressed through monetary policy, but must be balanced by country-specific fiscal policies. Whether this calls for coordination or not is a much debated issue, and has been typically investigated by looking at fiscal multipliers, as [Farhi and Werning \(2012\)](#) finds a greater output multiplier if government spending is financed by a foreign country rather than the home country. This would create a scope for a central EMU budget, as centrally financed government spending has larger effects than nationally financed government spending, but this is in a model without distortionary taxation, which is far from reality.

We focus on fiscal policy coordination with complete international financial markets and study the stabilization properties and the welfare implications of different fiscal policy scenarios. We study the effects of supply (productivity) and demand (preference) shocks hitting a single country under alternative fiscal policy specifications and compare uncoordinated fiscal policy (Pure Currency Union scenario), where each country chooses its government consumption, transfers and taxation, with fully coordinated fiscal policy (Full Fiscal Union scenario), where government consumption, transfers and taxation are chosen for each country by the union as a whole, with a consolidated budget. An intermediate case (Coordinated Currency Union scenario) shows the effects of coordinating fiscal policy without consolidating the budget.

We analyze the welfare gains from stabilization through variations in government consumption in the country hit by the shock, financed by the union as a whole, considering whether there is a scope for a fiscal capacity in the EMU to address asymmetric shocks to member countries, as a shock-absorption mechanism, as addressed in [Van Rompuy et al. \(2012\)](#). This was mentioned also in the more recent [Juncker et al. \(2015\)](#), where a Fiscal Union is seen as a Euro area-wide macroeconomic stabilization tool, over and above national fiscal policies needed to cushion country-specific shocks, which is thought to be key in avoiding pro-cyclical fiscal policies at all times. We define two welfare criteria and evaluate the welfare gains from a common macroeconomic stabilization function which can better deal with shocks that cannot be managed at the national level alone. We compare welfare under the Full Fiscal Union, the Coordinated Currency Union and the Pure Currency Union scenarios, bringing to policy conclusions for the proper macroeconomic management of a Currency Union. A future Fiscal Union will need a Eurozone treasury for collective decision-making on fiscal policy, which will also need to be accountable and legitimated democratically, but this is out of the scope of the present paper.

We follow the open economy approach of [Galí \(2009\)](#), in a two-country setting like in [Silveira \(2006\)](#),

but without taking the limit case of a small open economy. Our model follows the specifications of [Ferrero \(2009\)](#), which adapts the optimal approach of [Benigno and Woodford \(2004\)](#) to monetary and fiscal policy in a cashless closed economy without capital, where there are only distortionary taxes as sources of government revenue, to a two-country open-economy Currency Union setting. We follow also the Currency Union model of [Benigno \(2004\)](#), by adding a fiscal authority and including a government budget constraint, but without money in the utility function and concentrating on the cashless limit case. Our model adds home bias in consumption (or a degree of openness to international trade) and government transfers to households to the model in [Ferrero \(2009\)](#). The former allows for deviations from Purchasing Power Parity, while the latter is an additional instrument for fiscal policy.

As in [Ferrero \(2009\)](#), this model is structured to allow for spillovers from monetary to fiscal policy and viceversa, and from one country to another through country-specific fiscal policies. Nominal rigidities, in the form of staggered prices, generate real effects of monetary policy, while distortionary taxation generates non-Ricardian effects of fiscal policy. Monetary policy influences fiscal policy through its effect on the real value of debt, which depends on the interest rate and inflation. Fiscal policy influences monetary policy through its effect on prices and wages, which depend on the tax rates. This framework allows to study the interaction between country-specific fiscal policies, where in the absence of the nominal exchange rate as an automatic stabilizer, fiscal policies influence each other through their effects on output and the terms of trade.

The EMU is represented by two countries of different size forming a Currency Union. Each country has an independent Fiscal Authority, while the Currency Union shares a common Monetary Authority. The Central Bank sets the nominal interest rate for the whole Currency Union following an Inflation Targeting regime, where the target is on union-wide CPI inflation. This assumption reflects the current functioning of the ECB, whose primary objective of price stability is formulated as a situation in which the one-year increase in the CPI for the Eurozone is less than, but close to, 2%. Fiscal policy is designed following the Fiscal Compact Rules, by imposing that the Government Debt-to-GDP ratio is about 60% and that countries must adopt a balanced budget law in their national legislation. Following [Hjortsø \(2012\)](#), we add a stabilization rule for government consumption or alternatively for government transfers, but in a setting where distortionary taxes and government debt add the financing dimension to the fiscal policy problem. Governments choose the level of government consumption and transfers, which are financed by distortionary taxes on wage income and firm sales and by short-term government bonds. In particular:

- In a Pure Currency Union scenario governments choose the level of government consumption or transfers for domestic stabilization purposes by setting them alternatively to target the output gap, while

following in part an exogenous process, financed by a mix of distortionary tax rates, while balancing the budget.

- In a Coordinated Currency Union scenario governments choose the level of local government consumption or transfers for international stabilization purposes by setting them to target the net exports gap, while following in part an exogenous process, financed by a mix of distortionary tax rates, while balancing the budget.
- A Full Fiscal Union scenario uses a consolidated budget constraint to finance local government consumption or transfers for international stabilization purposes by setting them to target the net exports gap, while following in part an exogenous process, financed by a mix of distortionary tax rates, while balancing the consolidated budget, just like in the Currency Union case, but varying equally the tax rates across countries, so as to use union-wide resources to finance the government expenditure.

Our main results show that coordinating fiscal policy by targeting net exports, rather than output, produces more stable dynamics, and that consolidating government budget constraints across countries and moving tax rates jointly provides greater stabilization, while taxes on wage income are exponentially more distortionary than taxes on firm sales. Our policy prescriptions for the Eurozone are then to use fiscal policy to reduce international demand imbalances, either by stabilizing trade flows across countries or by creating some form of fiscal union or both, while avoiding the excessive use of labour taxes, in favour of sales taxes.

The remainder of the paper is structured as follows. Section 2 describes the general model and the fiscal policy scenarios of a Pure Currency Union, a Coordinated Currency Union and a Full Fiscal Union. Section 3 presents the calibration of the parameters and steady state stances of the model to two groups of countries in the EMU. Section 4 describes two welfare criteria and provides welfare rankings of the different fiscal policy scenarios. Section 5 provides numerical simulations under different scenarios, comparing different degrees of fiscal policy coordination, alternative fiscal policy instruments for stabilization and alternative government financing schemes. Section 6 shows the effects of fiscal shocks and describes fiscal policy multipliers and spillovers. Section 7 shows numerical simulations and welfare evaluations of the case for international goods as complements, rather than substitutes. Section 8 collects the main conclusions and provides possible extensions.

2 A Two-Country Currency Union Model

The world economy is composed of two countries, which form a Currency Union. Both economies are assumed to share identical preferences, technology and market structure, but may be subject to different

shocks, price rigidities, initial conditions and fiscal stances. The two countries are indexed by H and F for *Home* and *Foreign*. The world is populated by a continuum of infinitely-lived households of measure one, indexed by $i \in [0, 1]$. Each household owns a monopolistically competitive firm producing a differentiated good, indexed by $j \in [0, 1]$. The population on the segment $[0, h)$ belongs to country H while the population on the segment $[h, 1]$ belongs to country F . This means that the relative size of country H is $h \in [0, 1]$, while the relative size of country F is $1 - h$. This is true for both households and firms.

Firms set prices in a staggered fashion following Calvo (1983) and use only labour for production. There is no capital and no investment. Labour markets are competitive and internationally segmented. All goods are tradeable and the Law of One Price (LOP) holds for all single goods j . At the same time deviations from Purchasing Power Parity (PPP) may arise because of home bias in consumption. Financial markets are complete internationally, allowing households to trade a full set of one-period state-contingent claims across borders, other than purchase one-period risk-free bonds issued by the two countries' governments.

The world economy comprises also two Governments or Fiscal Authorities (one for each country), each one choosing government consumption and transfers to stabilize the economy, financed by distortionary taxes on wage income and firm sales and by short-term government bonds, and a Central Bank or Monetary Authority (one for the whole EMU), which sets the nominal interest rate targeting union-wide inflation.

We denote variables referred to the *Foreign* country with a star (*).

2.1 Households

In each country there is a continuum of households, which gain utility from private consumption and disutility from labour, consume goods produced in both countries with home bias, supply labour to domestic firms and collect profits from those firms. Households can trade a complete set of one-period state-contingent claims across borders and purchase one-period risk-free bonds issued by the two countries' governments, subject to their budget constraint.

Each household in country H , indexed by $i \in [0, h)$, and each household in country F , indexed by $i \in [h, 1]$, seeks to maximize respectively the present-value utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \xi_t U(C_t^i, N_t^i) \quad E_0 \sum_{t=0}^{\infty} \beta^t \xi_t^* U(C_t^{*i}, N_t^{*i}) \quad (2.1.1)$$

where $\beta \in [0, 1]$ is the common discount factor, which households use to discount future utility, ξ_t is a preference shock to *Home* households and ξ_t^* is a preference shock to *Foreign* households. These preference

shocks are assumed to follow the $AR(1)$ processes in logs:

$$\xi_t = (\xi_{t-1})^{\rho_\xi} e^{\varepsilon_t} \quad \text{and} \quad \xi_t^* = (\xi_{t-1}^*)^{\rho_\xi^*} e^{\varepsilon_t^*} \quad (2.1.2)$$

where $\rho_\xi \in [0, 1]$ and $\rho_\xi^* \in [0, 1]$ are measures of persistence of the shocks and ε_t is a zero mean white noise process. N_t^i denotes hours of labour supplied by households in country H , N_t^{*i} denotes hours of labour supplied by households in country F . C_t^i is a composite index for private consumption defined by¹:

$$C_t^i \equiv \left[(1 - \alpha)^{\frac{1}{\eta}} (C_{H,t}^i)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} (C_{F,t}^i)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \quad (2.1.3)$$

for households in country H , while the analogous index for households in country F , C_t^{*i} , is defined by:

$$C_t^{*i} \equiv \left[(1 - \alpha^*)^{\frac{1}{\eta}} (C_{H,t}^{*i})^{\frac{\eta-1}{\eta}} + \alpha^{*\frac{1}{\eta}} (C_{F,t}^{*i})^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \quad (2.1.4)$$

where $C_{H,t}^i$ is an index of consumption of domestic goods for households in country H , given by the constant elasticity of substitution (CES) function (also known as [Dixit and Stiglitz \(1977\)](#) aggregator function):

$$C_{H,t}^i \equiv \left(\left(\frac{1}{h} \right)^{\frac{1}{\varepsilon}} \int_0^h C_{H,t}^i(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad (2.1.5)$$

whereas, for households in country F the same index of consumption of domestic goods, $C_{H,t}^{*i}$, is given by:

$$C_{H,t}^{*i} \equiv \left(\left(\frac{1}{1-h} \right)^{\frac{1}{\varepsilon}} \int_h^1 C_{H,t}^{*i}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad (2.1.6)$$

where $j \in [0, 1]$ denotes a single good variety of the continuum of differentiated goods produced in the world economy. $C_{F,t}^i$ is an index of imported goods for households in country H , given by the analogous CES function:

$$C_{F,t}^i \equiv \left(\left(\frac{1}{1-h} \right)^{\frac{1}{\varepsilon}} \int_h^1 C_{F,t}^i(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad (2.1.7)$$

while the same index for imported goods for households in country F , $C_{F,t}^{*i}$, is given by:

$$C_{F,t}^{*i} \equiv \left(\left(\frac{1}{h} \right)^{\frac{1}{\varepsilon}} \int_0^h C_{F,t}^{*i}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad (2.1.8)$$

¹In the special case in which $\eta = 1$, the consumption index is given by $C_t^i = \left(\frac{C_{H,t}^i}{1-\alpha} \right)^{1-\alpha} \left(\frac{C_{F,t}^i}{\alpha} \right)^\alpha$. The same applies to the respective index for country F , which in the special case in which $\eta = 1$ is given by $C_t^{*i} = \left(\frac{C_{H,t}^{*i}}{1-\alpha^*} \right)^{1-\alpha^*} \left(\frac{C_{F,t}^{*i}}{\alpha^*} \right)^{\alpha^*}$.

The parameter $\varepsilon > 1$ measures the elasticity of substitution between varieties produced within a given country. The parameter $\eta > 0$ measures the substitutability between domestic and foreign goods (international trade elasticity). The parameter $\alpha \in [0, 1]$ is a measure of openness of the *Home* economy to international trade. Equivalently $(1 - \alpha)$ is a measure of the degree of home bias in consumption in country H . When α tends to zero the share of foreign goods in domestic consumption vanishes and the country ends up in autarky, consuming only domestic goods. If $1 - \alpha > h$ there is home bias in consumption in country H , because the share of consumption of domestic goods is greater than the share of production of domestic goods. The same applies to the *Foreign* parameter of openness to international trade $\alpha^* \in [0, 1]$ for country F , except for the fact that if $1 - \alpha^* > 1 - h$ there is home bias in consumption in country F , because the share of consumption of domestic goods in country F is greater than the share of production of domestic goods in country F .

We choose to specify additively separable period utility of the type with Constant Relative Risk Aversion (CRRA), so with constant elasticity of intertemporal substitution, and with constant elasticity of labour supply, which take the following form²:

$$U(C_t^i, N_t^i) \equiv \frac{(C_t^i)^{1-\sigma} - 1}{1-\sigma} - \frac{(N_t^i)^{1+\varphi}}{1+\varphi} \quad U(C_t^{*i}, N_t^{*i}) \equiv \frac{(C_t^{*i})^{1-\sigma} - 1}{1-\sigma} - \frac{(N_t^{*i})^{1+\varphi}}{1+\varphi} \quad (2.1.9)$$

for countries H and F respectively, where σ is the inverse of the elasticity of intertemporal substitution³ (it is also the Coefficient of Relative Risk Aversion (CRRA)), and φ is the inverse of the Frisch elasticity of labour supply⁴.

Households in country H maximize their present-value utility, equation 2.1.1, subject to the following sequence of budget constraints:

$$\int_0^h P_{H,t}(j) C_{H,t}^i(j) dj + \int_h^1 P_{F,t}(j) C_{F,t}^i(j) dj + D_t^i + B_t^i \leq \frac{D_{t-1}^i}{Q_{t-1,t}} + B_{t-1}^i(1 + i_{t-1}) + (1 - \tau_t^w) W_t N_t^i + T_t^i + \Gamma_t^i \quad (2.1.10)$$

for $t = 0, 1, 2, \dots$, where $P_{H,t}(j)$ is the price of domestic variety j , $P_{F,t}(j)$ is the price of variety j imported from country F , D_{t-1}^i is the portfolio of state-contingent claims purchased by the household in period $t - 1$, $Q_{t-1,t}$ is the stochastic discount factor, which is the same for households in both countries and represents the price of state-contingent claims or equivalently the inverse of the gross return on state-contingent claims,

²In the special case in which $\sigma = 1$, utility from private consumption is given by $U(C_t^i) = \ln C_t^i$ in country H and $U(C_t^{*i}) = \ln C_t^{*i}$ in country F .

³The elasticity of intertemporal substitution measures the responsiveness of consumption growth to changes in the real interest rate, which is the relative price of consumption between different dates, and is defined as the percent change in consumption growth divided by the percent change in the gross real interest rate.

⁴The Frisch elasticity of labour supply measures the extent to which labour supply responds to a change in the nominal wage, given a constant marginal utility of wealth, and is defined as the percent change in the supply of labour divided by the percent change in the nominal wage.

B_t^i are risk-free government bonds (of either or both governments) purchased by the household in period t , i_{t-1} is the nominal interest rate set by the central bank in period $t-1$, which is also the net return on government bonds, W_t is the nominal wage for households in country H , T_t^i denotes lump-sum transfers from the government to households, Γ_t^i denotes the share of profits net of taxes to households from ownership of firms and $\tau_t^w \in [0, 1]$ is a marginal tax rate on wage income paid by households to the government.

Households in country F analogously maximize their present-value utility, equation 2.1.1, subject to the following sequence of budget constraints:

$$\int_0^h P_{H,t}^*(j) C_{H,t}^{*i}(j) dj + \int_h^1 P_{F,t}^*(j) C_{F,t}^{*i}(j) dj + D_t^{*i} + B_t^{*i} \leq \frac{D_{t-1}^{*i}}{Q_{t-1,t}} + B_{t-1}^{*i} (1 + i_{t-1}) (1 - \tau_t^{*w}) W_t^* N_t^{*i} + T_t^{*i} + \Gamma_t^{*i} \quad (2.1.11)$$

for $t = 0, 1, 2, \dots$, where all the starred (*) variables are the *Foreign* equivalent of the unstarred ones explained above.

All variables are expressed in units of the union's currency. Last but not least, households in countries H and F respectively, are subject to the following solvency constraints, for all t , that prevent them from engaging in Ponzi-schemes:

$$\lim_{T \rightarrow \infty} E_t \{ Q_{t,T} D_T^i \} \geq 0 \quad \lim_{T \rightarrow \infty} E_t \{ Q_{t,T} D_T^{*i} \} \geq 0 \quad (2.1.12)$$

The first order conditions for the household's problem in country H are:

$$(C_t^i)^\sigma (N_t^i)^\varphi = (1 - \tau_t^w) \frac{W_t}{P_t} \quad (2.1.13)$$

which is the household's intratemporal optimality condition, and:

$$Q_{t,t+1} = \beta \frac{\xi_{t+1}}{\xi_t} \left(\frac{C_{t+1}^i}{C_t^i} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \quad (2.1.14)$$

which is the household's intertemporal optimality condition or Euler Equation, that holds for all states of nature at t and $t+1$, and:

$$\frac{1}{1 + i_t} = E_t \{ Q_{t,t+1} \} \quad (2.1.15)$$

which is the no-arbitrage condition between riskless government bonds and state-contingent claims.

Aggregating the intratemporal optimality condition, equation 2.1.13, yields the aggregate labour supply

equation for households in country H :

$$N_t = (h)^{1+\frac{\sigma}{\varphi}} (C_t)^{-\frac{\sigma}{\varphi}} \left[(1 - \tau_t^w) \frac{W_t}{P_t} \right]^{\frac{1}{\varphi}} \quad (2.1.16)$$

where N_t is aggregate labour supply and C_t is aggregate consumption for households in country H , defined by:

$$N_t \equiv \int_0^h N_t^i di = h N_t^i \quad C_t \equiv \int_0^h C_t^i di = h C_t^i \quad (2.1.17)$$

while aggregating the intertemporal optimality condition for households in country H , equation 2.1.14, taking conditional expectations on both sides of the equation and using the no-arbitrage condition between government bonds and state-contingent claims, yields:

$$\frac{1}{1+i_t} = E_t\{\mathcal{Q}_{t,t+1}\} = \beta E_t \left\{ \frac{\xi_{t+1}}{\xi_t} \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{1}{\Pi_{t+1}} \right\} \quad (2.1.18)$$

where $\frac{1}{1+i_t} = E_t\{\mathcal{Q}_{t,t+1}\}$ is the price of a one-period riskless government bond paying off one unit of the union's currency in $t+1$ and $\Pi_{t+1} \equiv \frac{P_{t+1}}{P_t}$ is gross CPI inflation in country H .

Specular equations for country F can be obtained following the same procedure, where the results are the same, but with all the relevant starred (*) variables instead of the unstarred ones.

The first order conditions for the household's problem in country F are:

$$(C_t^{*i})^\sigma (N_t^{*i})^\varphi = (1 - \tau_t^{*w}) \frac{W_t^*}{P_t^*} \quad (2.1.19)$$

which is the household's intratemporal optimality condition, and:

$$\mathcal{Q}_{t,t+1} = \beta \frac{\xi_{t+1}^*}{\xi_t^*} \left(\frac{C_{t+1}^{*i}}{C_t^{*i}} \right)^{-\sigma} \frac{P_t^*}{P_{t+1}^*} \quad (2.1.20)$$

which is the household's intertemporal optimality condition or Euler Equation, that holds for all states of nature at t and $t+1$, and:

$$\frac{1}{1+i_t} = E_t\{\mathcal{Q}_{t,t+1}\} \quad (2.1.21)$$

which is the no-arbitrage condition between riskless government bonds and state-contingent claims.

The aggregate labour supply equation for households in country F is given by:

$$N_t^* = (1-h)^{1+\frac{\sigma}{\varphi}} (C_t^*)^{-\frac{\sigma}{\varphi}} \left[(1 - \tau_t^{*w}) \frac{W_t^*}{P_t^*} \right]^{\frac{1}{\varphi}} \quad (2.1.22)$$

where N_t^* is aggregate labour supply and C_t^* is aggregate consumption for households in country F , defined by:

$$N_t^* \equiv \int_h^1 N_t^{*i} di = (1-h)N_t^{*i} \quad C_t^* \equiv \int_h^1 C_t^{*i} di = (1-h)C_t^{*i} \quad (2.1.23)$$

while taking conditional expectations on both sides of the aggregate intertemporal optimality condition and using the no-arbitrage condition between government bonds and state-contingent claims yields:

$$\frac{1}{1+i_t} = E_t\{\mathcal{Q}_{t,t+1}\} = \beta E_t\left\{\frac{\xi_{t+1}^*}{\xi_t^*} \left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\sigma} \frac{1}{\Pi_{t+1}^*}\right\} \quad (2.1.24)$$

where $\frac{1}{1+i_t} = E_t\{\mathcal{Q}_{t,t+1}\}$ is the price of a one-period riskless government bond paying off one unit of the union's currency in $t+1$, which is the same for both countries in the currency union, and $\Pi_{t+1}^* \equiv \frac{P_{t+1}^*}{P_t^*}$ is gross CPI inflation in country F .

Aggregating the budget constraints of households in countries H and F respectively and considering that in optimality they hold with equality yields:

$$P_t C_t + D_t + B_t = (1+i_{t-1})(D_{t-1} + B_{t-1}) + (1-\tau_t^w)W_t N_t + T_t + \Gamma_t \quad (2.1.25)$$

$$P_t^* C_t^* + D_t^* + B_t^* = (1+i_{t-1})(D_{t-1}^* + B_{t-1}^*) + (1-\tau_t^{*w})W_t^* N_t^* + T_t^* + \Gamma_t^* \quad (2.1.26)$$

where aggregate contingent claims and aggregate bonds are respectively defined as:

$$D_t \equiv \int_0^h D_t^i di = hD_t^i \quad D_t^* \equiv \int_h^1 D_t^{*i} di = (1-h)D_t^{*i} \quad (2.1.27)$$

$$B_t \equiv \int_0^h B_t^i di = hB_t^i \quad B_t^* \equiv \int_h^1 B_t^{*i} di = (1-h)B_t^{*i} \quad (2.1.28)$$

while aggregate transfers and profits are respectively defined as:

$$T_t \equiv \int_0^h T_t^i di = hT_t^i \quad T_t^* \equiv \int_h^1 T_t^{*i} di = (1-h)T_t^{*i} \quad (2.1.29)$$

$$\Gamma_t \equiv \int_0^h \Gamma_t^i di = h\Gamma_t^i \quad \Gamma_t^* \equiv \int_h^1 \Gamma_t^{*i} di = (1-h)\Gamma_t^{*i} \quad (2.1.30)$$

and aggregate consumption and labour supply are as previously defined.

P_t is the Consumer Price Index (CPI) for country H , given by⁵:

$$P_t \equiv [(1-\alpha)(P_{H,t})^{1-\eta} + \alpha(P_{F,t})^{1-\eta}]^{\frac{1}{1-\eta}} \quad (2.1.31)$$

⁵The Consumer Price Index (CPI) for country H , P_t , can be found by minimizing a given expenditure on both domestic

while P_t^* is the Consumer Price Index (CPI) for country F , given by:

$$P_t^* \equiv [(1 - \alpha^*)(P_{H,t}^*)^{1-\eta} + \alpha^*(P_{F,t}^*)^{1-\eta}]^{\frac{1}{1-\eta}} \quad (2.1.32)$$

where $P_{H,t}$ is the domestic price index or Producer Price Index (PPI) in country H and $P_{F,t}$ is a price index for goods imported from country F , respectively defined by:

$$P_{H,t} \equiv \left(\frac{1}{h} \int_0^h P_{H,t}(j)^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}} \quad (2.1.33)$$

$$P_{F,t} \equiv \left(\frac{1}{1-h} \int_h^1 P_{F,t}(j)^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}} \quad (2.1.34)$$

while $P_{H,t}^*$ is the domestic price index or Producer Price Index (PPI) in country F and $P_{F,t}^*$ is a price index for goods imported from country H , respectively defined by:

$$P_{H,t}^* \equiv \left(\frac{1}{1-h} \int_h^1 P_{H,t}^*(j)^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}} \quad (2.1.35)$$

$$P_{F,t}^* \equiv \left(\frac{1}{h} \int_0^h P_{F,t}^*(j)^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}} \quad (2.1.36)$$

Since one-period state-contingent claims can be traded freely between households within and across borders, they are in zero international net supply, so that the market clearing condition for these assets in every period t is consequently given by:

$$\int_0^h D_t^i di + \int_h^1 D_t^{*i} di = hD_t^i + (1-h)D_t^{*i} = D_t + D_t^* = 0 \quad (2.1.37)$$

Adding up the two budget constraints and imposing the market clearing condition for state-contingent claims yields an equation linking consumption in the two countries:

$$\begin{aligned} P_t C_t + B_t + P_t^* C_t^* + B_t^* &= (1 + i_{t-1}) (B_{t-1} + B_{t-1}^*) \\ &+ (1 - \tau_t^w) W_t N_t + T_t + \Gamma_t + (1 - \tau_t^{*w}) W_t^* N_t^* + T_t^* + \Gamma_t^* \end{aligned} \quad (2.1.38)$$

and foreign goods under the constraint that equation 2.1.3 equals one:

$$\min_{C_{H,t}^i, C_{F,t}^i} P_{H,t} C_{H,t}^i + P_{F,t} C_{F,t}^i \quad s.t. \quad \left[(1 - \alpha)^{\frac{1}{\eta}} (C_{H,t}^i)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} (C_{F,t}^i)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} = 1$$

The same applies to the respective Consumer Price Index (CPI) for country F , P_t^* . In the particular case in which $\eta = 1$, the CPI in country H takes the form $P_t = (P_{H,t})^{1-\alpha} (P_{F,t})^\alpha$, while the CPI in country F takes the form $P_t^* = (P_{H,t}^*)^{1-\alpha^*} (P_{F,t}^*)^{\alpha^*}$.

2.2 International Identities and Assumptions

Several international identities and assumptions need to be spelled out in order to link the *Home* economy to the *Foreign* one and to be able to close the model.

The *terms of trade* are defined as the price of foreign goods in terms of home goods, for households in country H and in country F , and are given respectively by:

$$\mathcal{S}_t \equiv \frac{P_{F,t}}{P_{H,t}} \quad \text{and} \quad \mathcal{S}_t^* \equiv \frac{P_{F,t}^*}{P_{H,t}^*} \quad (2.2.1)$$

Although deviations from *Purchasing Power Parity* (PPP) may arise because of home bias in consumption, we assume that the *Law of One Price* (LOP) holds for every single good j , which implies:

$$P_{H,t}(j) = P_{F,t}^*(j) \quad \text{and} \quad P_{F,t}(j) = P_{H,t}^*(j) \quad (2.2.2)$$

for all $j \in [0, 1]$, where $P_{H,t}(j)$ (or $P_{F,t}(j)$ for goods imported from country F) is the price of good j in country H and $P_{F,t}^*(j)$ (or $P_{H,t}^*(j)$ for goods produced in country F) is the price of good j in country F in terms of the union's currency. Plugging the previous expressions into the definitions of $P_{H,t}$ and $P_{F,t}$ respectively yields:

$$P_{H,t} = P_{F,t}^* \quad \text{and} \quad P_{F,t} = P_{H,t}^* \quad (2.2.3)$$

Combining the previous result with the definition of the terms of trade for countries H and F yields:

$$\mathcal{S}_t = \frac{P_{F,t}}{P_{H,t}} = \frac{P_{H,t}^*}{P_{F,t}^*} = \frac{1}{\mathcal{S}_t^*} \quad (2.2.4)$$

Dividing the CPIs in countries H and F by the relative price indices for domestic and imported goods and combining them with the definition of the terms of trade yields the following relationships⁶:

$$\frac{P_t}{P_{H,t}} = [1 - \alpha + \alpha(\mathcal{S}_t)^{1-\eta}]^{\frac{1}{1-\eta}} \quad (2.2.5)$$

$$\frac{P_t}{P_{F,t}} = \frac{[1 - \alpha + \alpha(\mathcal{S}_t)^{1-\eta}]^{\frac{1}{1-\eta}}}{\mathcal{S}_t} \quad (2.2.6)$$

⁶In the special case in which $\eta = 1$ these ratios take the form:

$$\begin{aligned} \frac{P_t}{P_{H,t}} &= \left(\frac{P_{F,t}}{P_{H,t}} \right)^\alpha = \mathcal{S}_t^\alpha \quad \text{and} \quad \frac{P_t}{P_{F,t}} = \left(\frac{P_{H,t}}{P_{F,t}} \right)^{1-\alpha} = \mathcal{S}_t^{\alpha-1} \\ \frac{P_t^*}{P_{H,t}^*} &= \left(\frac{P_{F,t}^*}{P_{H,t}^*} \right)^{\alpha^*} = (\mathcal{S}_t^*)^{\alpha^*} = (\mathcal{S}_t)^{-\alpha^*} \quad \text{and} \quad \frac{P_t^*}{P_{F,t}^*} = \left(\frac{P_{H,t}^*}{P_{F,t}^*} \right)^{1-\alpha^*} = (\mathcal{S}_t^*)^{\alpha^*-1} = (\mathcal{S}_t)^{1-\alpha^*} \end{aligned}$$

$$\frac{P_t^*}{P_{H,t}^*} = [1 - \alpha^* + \alpha^*(\mathcal{S}_t^*)^{1-\eta}]^{\frac{1}{1-\eta}} = [1 - \alpha^* + \alpha^*(\mathcal{S}_t)^{\eta-1}]^{\frac{1}{1-\eta}} \quad (2.2.7)$$

$$\frac{P_t^*}{P_{F,t}^*} = \frac{[1 - \alpha^* + \alpha^*(\mathcal{S}_t^*)^{1-\eta}]^{\frac{1}{1-\eta}}}{\mathcal{S}_t^*} = \mathcal{S}_t [1 - \alpha^* + \alpha^*(\mathcal{S}_t)^{\eta-1}]^{\frac{1}{1-\eta}} \quad (2.2.8)$$

Computing the first price ratio for periods t and $t-1$ and dividing one by the other yields:

$$\frac{P_t}{P_{H,t}} \frac{P_{H,t-1}}{P_{t-1}} = \frac{[1 - \alpha + \alpha(\mathcal{S}_t)^{1-\eta}]^{\frac{1}{1-\eta}}}{[1 - \alpha + \alpha(\mathcal{S}_{t-1})^{1-\eta}]^{\frac{1}{1-\eta}}} \quad (2.2.9)$$

which rearranged yields a relationship between PPI inflation and CPI inflation in country H ⁷:

$$\Pi_t = \Pi_{H,t} \left[\frac{1 - \alpha + \alpha(\mathcal{S}_t)^{1-\eta}}{1 - \alpha + \alpha(\mathcal{S}_{t-1})^{1-\eta}} \right]^{\frac{1}{1-\eta}} \quad (2.2.10)$$

The same relationship can be derived for country F and will be given by:

$$\Pi_t^* = \Pi_{H,t}^* \left[\frac{1 - \alpha^* + \alpha^*(\mathcal{S}_t^*)^{\eta-1}}{1 - \alpha^* + \alpha^*(\mathcal{S}_{t-1})^{\eta-1}} \right]^{\frac{1}{1-\eta}} \quad (2.2.11)$$

Dividing the terms of trade in period t by the terms of trade in period $t-1$ yields a relationship showing the evolution of the terms of trade over time:

$$\frac{\mathcal{S}_t}{\mathcal{S}_{t-1}} = \frac{\Pi_{F,t}}{\Pi_{H,t}} = \frac{\Pi_{H,t}^*}{\Pi_{H,t}} \implies \mathcal{S}_t = \frac{\Pi_{H,t}^*}{\Pi_{H,t}} \mathcal{S}_{t-1} \quad (2.2.12)$$

as a function of PPI inflation in both countries H and F .

The *Real Exchange Rate* between the *Home* country and country F is the ratio of the two countries' CPIs, expressed both in terms of the union's currency, and is defined by:

$$Q_t \equiv \frac{P_t^*}{P_t} \quad (2.2.13)$$

Combining the previous results with the definition of the real exchange rate yields a relationship between

⁷In the special case in which $\eta = 1$ the price ratios take the form shown above and the relationships between PPI inflation and CPI inflation in countries H and F respectively take the form:

$$\Pi_t = \Pi_{H,t} \left(\frac{\mathcal{S}_t}{\mathcal{S}_{t-1}} \right)^\alpha \quad \text{and} \quad \Pi_t^* = \Pi_{H,t}^* \left(\frac{\mathcal{S}_t^*}{\mathcal{S}_{t-1}^*} \right)^{\alpha^*} = \Pi_{H,t}^* \left(\frac{\mathcal{S}_{t-1}}{\mathcal{S}_t} \right)^{\alpha^*}$$

the real exchange rate and the terms of trade⁸:

$$Q_t = \frac{P_t^*}{P_{F,t}^*} \frac{P_{H,t}}{P_t} = \frac{[1 - \alpha^* + \alpha^*(S_t^*)^{1-\eta}]^{\frac{1}{1-\eta}}}{S_t^* [1 - \alpha + \alpha(S_t)^{1-\eta}]^{\frac{1}{1-\eta}}} = S_t \left[\frac{1 - \alpha^* + \alpha^*(S_t)^{\eta-1}}{1 - \alpha + \alpha(S_t)^{1-\eta}} \right]^{\frac{1}{1-\eta}} \quad (2.2.14)$$

where the difference between the real exchange rate and the terms of trade is given by the degree of openness of the two countries and the international trade elasticity. If the countries both have complete home bias ($\alpha = \alpha^* = 0$), then they are in autarky and the real exchange rate is exactly equal to the terms of trade, because the CPI and PPI are the same in each country.

2.3 International Risk-Sharing

The no-arbitrage condition for state-contingent claims and the assumption of complete markets implies that the price of the state-contingent claims must be the same for households in both countries. This in turn implies that we can link the consumption of households in the two countries by equating their Euler Equations through their Stochastic Discount Factor, which yields an international risk-sharing condition.

The risk sharing condition linking consumption of households in country H to consumption of households in country F , through their Euler Equations, is given by:

$$C_{t+1}^i = \left(\frac{\xi_{t+1}}{\xi_t} \frac{\xi_t^*}{\xi_{t+1}^*} \frac{Q_{t+1}}{Q_t} \right)^{\frac{1}{\sigma}} \frac{C_t^i}{C_t^{*i}} C_{t+1}^{*i} \quad (2.3.1)$$

By repeated substitution of *Home* consumption backward in time and moving backwards one period, the previous equation reduces to one linking the consumption of households in the two countries as a function of initial conditions, the real exchange rate and preference shocks:

$$C_t^i = \left(\frac{\xi_0^*}{\xi_0} \frac{1}{Q_0} \right)^{\frac{1}{\sigma}} \left(\frac{C_0^i}{C_0^{*i}} \right) \left(\frac{\xi_t}{\xi_t^*} Q_t \right)^{\frac{1}{\sigma}} C_t^{*i} \quad (2.3.2)$$

By assuming symmetric initial conditions for households in countries H and F , the previous expression reduces to:

$$C_t^i = \left(\frac{\xi_t}{\xi_t^*} Q_t \right)^{\frac{1}{\sigma}} C_t^{*i} \quad (2.3.3)$$

where household consumption in the two countries differs only in the presence of asymmetric preference shocks and of deviations from purchasing power parity (when the real exchange rate is different from one).

⁸In the special case in which $\eta = 1$ the real exchange rate takes the form:

$$Q_t = \left(\frac{P_{F,t}}{P_{H,t}} \right)^{1-\alpha-\alpha^*} = S_t^{1-\alpha-\alpha^*}$$

Aggregating the previous condition across households in each country yields:

$$C_t = \frac{h}{1-h} \left(\frac{\xi_t}{\xi_t^*} Q_t \right)^{\frac{1}{\sigma}} C_t^* \quad (2.3.4)$$

where C_t and C_t^* are aggregate consumption for households in countries H and F respectively, as previously defined, and the difference in aggregate consumption across countries is also given by the relative size of the two countries.

Substituting the real exchange rate with equation 2.2.14 yields an international risk-sharing condition linking aggregate consumption in the two countries as a function of the terms of trade, the degree of openness of the two countries and the international trade elasticity, other than preference shocks and country size⁹:

$$C_t = \frac{h}{1-h} \left[\frac{\xi_t}{\xi_t^*} \mathcal{S}_t \left(\frac{1 - \alpha^* + \alpha^* (\mathcal{S}_t)^{\eta-1}}{1 - \alpha + \alpha (\mathcal{S}_t)^{1-\eta}} \right)^{\frac{1}{1-\eta}} \right]^{\frac{1}{\sigma}} C_t^* \quad (2.3.6)$$

2.4 Firms

In country H there is a continuum of firms indexed by $j \in [0, h]$, while in country F there is a continuum of firms, indexed by $j \in [h, 1]$, each producing a differentiated good with the same technology represented respectively by the following production functions:

$$Y_t(j) = A_t N_t(j) \quad Y_t^*(j) = A_t^* N_t^*(j) \quad (2.4.1)$$

where A_t and A_t^* represent the level of technology in countries H and F , respectively, which evolve exogenously over time following the $AR(1)$ processes in logs:

$$A_t = (A_{t-1})^{\rho_a} e^{\varepsilon_t} \quad \text{and} \quad A_t^* = (A_{t-1}^*)^{\rho_a^*} e^{\varepsilon_t} \quad (2.4.2)$$

where $\rho_a \in [0, 1]$ and $\rho_a^* \in [0, 1]$ are measures of persistence of the shocks and ε_t is a zero mean white noise process.

From the production functions we can derive labour demand for individual firms in countries H and F and the respective nominal and real marginal costs of production, which are equal across firms in each

⁹In the special case in which $\eta = 1$ the international risk-sharing condition takes the form:

$$C_t = \frac{h}{1-h} \left(\frac{\xi_t}{\xi_t^*} \mathcal{S}_t^{1-\alpha-\alpha^*} \right)^{\frac{1}{\sigma}} C_t^* \quad (2.3.5)$$

country and are given by¹⁰:

$$N_t(j) = \frac{Y_t(j)}{A_t} \implies MC_t^n = \frac{W_t}{A_t} \implies MC_t = \frac{W_t}{A_t P_{H,t}} \quad (2.4.3)$$

$$N_t^*(j) = \frac{Y_t^*(j)}{A_t^*} \implies MC_t^{*n} = \frac{W_t^*}{A_t^*} \implies MC_t^* = \frac{W_t^*}{A_t^* P_{H,t}^*} \quad (2.4.4)$$

Aggregating individual labour demand across firms in each country yields the aggregate labour demand for countries H and F , respectively given by¹¹:

$$N_t \equiv \int_0^h N_t(j) dj = \int_0^h \frac{Y_t(j)}{A_t} dj = \frac{Y_t}{A_t} \int_0^h \frac{1}{h} \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} dj = \frac{Y_t}{A_t} d_t \quad (2.4.5)$$

$$N_t^* \equiv \int_h^1 N_t^*(j) dj = \int_h^1 \frac{Y_t^*(j)}{A_t^*} dj = \frac{Y_t^*}{A_t^*} \int_h^1 \frac{1}{1-h} \left(\frac{P_{H,t}^*(j)}{P_{H,t}^*} \right)^{-\varepsilon} dj = \frac{Y_t^*}{A_t^*} d_t^* \quad (2.4.6)$$

where Y_t and Y_t^* are aggregate output in countries H and F , respectively given by:

$$Y_t \equiv \left(\left(\frac{1}{h} \right)^{\frac{1}{\varepsilon}} \int_0^h Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad Y_t^* \equiv \left(\left(\frac{1}{1-h} \right)^{\frac{1}{\varepsilon}} \int_h^1 Y_t^*(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad (2.4.7)$$

and where the terms:

$$d_t \equiv \int_0^h \frac{1}{h} \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} dj \quad \text{and} \quad d_t^* \equiv \int_h^1 \frac{1}{1-h} \left(\frac{P_{H,t}^*(j)}{P_{H,t}^*} \right)^{-\varepsilon} dj \quad (2.4.8)$$

represent relative price dispersion across firms in each country. In steady state and in a flexible price equilibrium these relative price dispersions are equal to one.

Firm j 's period t profits net of taxes in country H are given by:

$$\Gamma_t(j) = (1 - \tau_t^s) P_{H,t}(j) Y_t(j) - W_t N_t(j) \quad (2.4.9)$$

where τ_t^s is the marginal tax rate on firm sales in country H . Substituting in labour demand, marginal costs, substituting the output of firm j with the demand function for output, integrating over all $j \in [0, h]$, using

¹⁰Individual labour demand can be derived directly from the production function by dividing both sides by A_t , while nominal marginal costs can be derived by minimizing costs for firms subject to the production function, or labour demand:

$$\min_{Y_t(j)} W_t N_t(j) \quad \text{s.t.} \quad Y_t(j) = A_t N_t(j)$$

Real marginal costs are simply nominal marginal costs divided by the PPI, which is the relevant price for firms.

¹¹The results are obtained by substituting in the demand functions for output in each country, given by:

$$Y_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} \frac{Y_t}{h} \quad \text{and} \quad Y_t^*(j) = \left(\frac{P_{H,t}^*(j)}{P_{H,t}^*} \right)^{-\varepsilon} \frac{Y_t^*}{1-h}$$

the definition of $P_{H,t}$, and substituting in price dispersion yields:

$$\Gamma_t = (1 - \tau_t^s)P_{H,t}Y_t - P_{H,t}MC_tY_t d_t = P_{H,t}Y_t(1 - \tau_t^s - MC_t d_t) \quad (2.4.10)$$

Analogous results can be obtained for aggregate profits net of taxes in country F :

$$\Gamma_t^* \equiv \int_h^1 \Gamma_t^*(j) dj = (1 - \tau_t^{*s})P_{H,t}^*Y_t^* - P_{H,t}^*MC_t^*Y_t^* d_t^* = P_{H,t}^*Y_t^*(1 - \tau_t^{*s} - MC_t^* d_t^*) \quad (2.4.11)$$

where τ_t^{*s} is the marginal tax rate on firm sales in country F , and where firm j 's period t profits net of taxes in country F are given by:

$$\Gamma_t^*(j) = (1 - \tau_t^{*s})P_{H,t}^*(j)Y_t^*(j) - W_t^*N_t^*(j) = Y_t^*(j)[(1 - \tau_t^{*s})P_{H,t}^*(j) - MC_t^{*n}] \quad (2.4.12)$$

Here we can see how aggregate profits are a function of the PPI, aggregate output, marginal cost and price dispersion for both countries H and F , other than the tax rate on firm sales, while nominal aggregate wage income is shown to be a function of some of the same variables and equal to:

$$W_t N_t = W_t \frac{Y_t}{A_t} d_t = MC_t^n Y_t d_t = P_{H,t} Y_t MC_t d_t \quad (2.4.13)$$

in country H , and:

$$W_t^* N_t^* = W_t^* \frac{Y_t^*}{A_t^*} d_t^* = MC_t^{*n} Y_t^* d_t^* = P_{H,t}^* Y_t^* MC_t^* d_t^* \quad (2.4.14)$$

in country F .

Following [Calvo \(1983\)](#), each firm in country H may reset its price with probability $1 - \theta$ in any given period. Thus, each period a fraction $1 - \theta$ of randomly selected firms reset their price, while a fraction θ keep their prices unchanged. As a result, the average duration of a price in country H is given by $(1 - \theta)^{-1}$, and θ can be seen as a natural index of price stickiness for country H . In country F each firm may reset its price with probability $1 - \theta^*$ in any given period. Thus, each period a fraction $1 - \theta^*$ of randomly selected firms reset their price, while a fraction θ^* keep their prices unchanged. As a result, the average duration of a price in country F is given by $(1 - \theta^*)^{-1}$, and θ^* can be seen as a natural index of price stickiness for country F . This allows for the two countries to have different degrees of price rigidity.

A firm in country H re-optimizing in period t will choose the price $\bar{P}_{H,t}$ that maximizes the current market value of the profits net of taxes generated while that price remains effective. Formally, it solves the

problem:

$$\max_{\bar{P}_{H,t}} \sum_{k=0}^{\infty} \theta^k E_t \left\{ \mathcal{Q}_{t,t+k} Y_{t+k|t}(j) \left[(1 - \tau_{t+k}^s) \bar{P}_{H,t} - MC_{t+k}^n \right] \right\} \quad (2.4.15)$$

subject to the sequence of demand constraints ¹²:

$$Y_{t+k|t}(j) = \left(\frac{\bar{P}_{H,t}}{P_{H,t+k}} \right)^{-\varepsilon} \frac{Y_{t+k}}{h} \quad (2.4.16)$$

for $k = 0, 1, 2, \dots$, where $\mathcal{Q}_{t,t+k}$ is the household's stochastic discount factor in country H for discounting k -period ahead nominal payoffs from ownership of firms, defined by:

$$\mathcal{Q}_{t,t+k} = \beta^k \frac{\xi_{t+k}}{\xi_t} \left(\frac{C_{t+k}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+k}} \quad (2.4.17)$$

for $k = 0, 1, 2, \dots$, and where $Y_{t+k|t}(j)$ is the output in period $t + k$ for firm j which last reset its price in period t .

Analogously, a firm in country F re-optimizing in period t will choose the price $\bar{P}_{H,t}^*$ that maximizes the current market value of the profits net of taxes generated while that price remains effective. Formally, it solves the problem:

$$\max_{\bar{P}_{H,t}^*} \sum_{k=0}^{\infty} \theta^{*k} E_t \left\{ \mathcal{Q}_{t,t+k} Y_{t+k|t}^*(j) \left[(1 - \tau_{t+k}^{*s}) \bar{P}_{H,t}^* - MC_{t+k}^{*n} \right] \right\} \quad (2.4.18)$$

subject to the sequence of demand constraints:

$$Y_{t+k|t}^*(j) = \left(\frac{\bar{P}_{H,t}^*}{P_{H,t+k}^*} \right)^{-\varepsilon} \frac{Y_{t+k}^*}{1 - h} \quad (2.4.19)$$

for $k = 0, 1, 2, \dots$, where $\mathcal{Q}_{t,t+k}$ is the household's stochastic discount factor in country F for discounting k -period ahead nominal payoffs from ownership of firms, defined by:

$$\mathcal{Q}_{t,t+k} = \beta^k \frac{\xi_{t+k}^*}{\xi_t^*} \left(\frac{C_{t+k}^*}{C_t^*} \right)^{-\sigma} \frac{P_t^*}{P_{t+k}^*} \quad (2.4.20)$$

for $k = 0, 1, 2, \dots$, which is equal to the stochastic discount factor for households in country H , and where $Y_{t+k|t}^*(j)$ is the output in period $t + k$ for firm j which last reset its price in period t .

The optimal relative price chosen by firms in country H can be expressed as a function of only aggregate

¹²The derivation of the demand function for firms is much like the derivation of the demand function for consumption goods, except for the timing of price setting, which implies that $P_{H,t+k}(j) = \bar{P}_{H,t}(j)$ with probability θ^k for $k = 0, 1, 2, \dots$, and the fact that all firms are the same and so they set the same price in any given period, which allows us to drop the j index.

variables:

$$\bar{P}_{H,t} = \frac{\varepsilon}{\varepsilon - 1} \frac{\sum_{k=0}^{\infty} (\beta\theta)^k E_t \left\{ \frac{\xi_{t+k}(C_{t+k})^{-\sigma}}{P_{t+k}} \frac{Y_{t+k}}{(P_{H,t+k})^{-\varepsilon}} MC_{t+k}^n \right\}}{\sum_{k=0}^{\infty} (\beta\theta)^k E_t \left\{ \frac{\xi_{t+k}(C_{t+k})^{-\sigma}}{P_{t+k}} \frac{Y_{t+k}}{(P_{H,t+k})^{-\varepsilon}} (1 - \tau_{t+k}^s) \right\}} \quad (2.4.21)$$

Notice that in the zero inflation steady state and in the flexible price equilibrium the previous equation simplifies to:

$$\bar{P}_H = \frac{\varepsilon}{(\varepsilon - 1)(1 - \tau^s)} MC^n \quad (2.4.22)$$

where MC^n is the nominal marginal cost in steady state and in the flexible price equilibrium in country H , and where the optimal price is shown to be set as a markup over nominal marginal costs.

Analogous results hold for firms in country F . The optimal price chosen by firms in country F can be expressed as a function of only aggregate variables:

$$\bar{P}_{H,t}^* = \frac{\varepsilon}{\varepsilon - 1} \frac{\sum_{k=0}^{\infty} (\beta\theta^*)^k E_t \left\{ \frac{\xi_{t+k}^*(C_{t+k}^*)^{-\sigma}}{P_{t+k}^*} \frac{Y_{t+k}^*}{(P_{H,t+k}^*)^{-\varepsilon}} MC_{t+k}^{*n} \right\}}{\sum_{k=0}^{\infty} (\beta\theta^*)^k E_t \left\{ \frac{\xi_{t+k}^*(C_{t+k}^*)^{-\sigma}}{P_{t+k}^*} \frac{Y_{t+k}^*}{(P_{H,t+k}^*)^{-\varepsilon}} (1 - \tau_{t+k}^{*s}) \right\}} \quad (2.4.23)$$

In the zero inflation steady state and in the flexible price equilibrium the previous equation simplifies to:

$$\bar{P}_H^* = \frac{\varepsilon}{(\varepsilon - 1)(1 - \tau^{*s})} MC^{*n} \quad (2.4.24)$$

where MC^{*n} is the nominal marginal cost in steady state and in the flexible price equilibrium in country F , and where the optimal price is shown to be set as a markup over nominal marginal costs.

2.5 Net Exports, Net Foreign Assets and the Balance of Payments

Net Exports are defined as domestic production minus domestic consumption, which is equal to exports minus imports, and for country H are given by:

$$NX_t \equiv P_{H,t}Y_t - P_tC_t - P_{H,t}G_t \quad (2.5.1)$$

In real terms (divided by $P_{H,t}$), Net Exports for country H can be written as:

$$\widetilde{NX}_t = Y_t - \frac{P_t}{P_{H,t}}C_t - G_t = Y_t - [1 - \alpha + \alpha(\mathcal{S}_t)^{1-\eta}]^{\frac{1}{1-\eta}} C_t - G_t \quad (2.5.2)$$

where net exports are shown to be a function of the country's degree of openness and the terms of trade, other than domestic production and public and private domestic consumption.

Since exports for country H are imports for country F and viceversa, then net exports are in zero

international net supply: $NX_t + NX_t^* = 0$. In real terms: $\widetilde{NX}_t + S_t \widetilde{NX}_t^* = 0$. This implies the following relationship between output in the two countries:

$$Y_t + S_t Y_t^* = [1 - \alpha + \alpha(\mathcal{S}_t)^{1-\eta}]^{\frac{1}{1-\eta}} C_t + S_t [1 - \alpha^* + \alpha^*(\mathcal{S}_t)^{\eta-1}]^{\frac{1}{1-\eta}} C_t^* + G_t + S_t G_t^* \quad (2.5.3)$$

Net Foreign Assets are given by the sum of private and public assets held abroad, and for countries H and F are given respectively by:

$$NFA_t \equiv D_t + B_t - B_t^G \quad \text{and} \quad NFA_t^* \equiv D_t^* + B_t^* - B_t^{*G} \quad (2.5.4)$$

In real terms (divided by $P_{H,t}$ and $P_{H,t}^*$ respectively), Net Foreign Assets for countries H and F can be written respectively as:

$$\widetilde{NFA}_t \equiv \tilde{D}_t + \tilde{B}_t - \tilde{B}_t^G \quad \text{and} \quad \widetilde{NFA}_t^* \equiv \tilde{D}_t^* + \tilde{B}_t^* - \tilde{B}_t^{*G} \quad (2.5.5)$$

Since foreign assets for country H are domestic assets for country F , then net foreign assets are in zero international net supply: $NFA_t + NFA_t^* = 0$. In real terms: $\widetilde{NFA}_t + S_t \widetilde{NFA}_t^* = 0$.

The *Balance of Payments* is given by net exports plus interest accrued on net foreign assets:

$$BP_t \equiv NX_t + i_{t-1} NFA_{t-1} \quad (2.5.6)$$

In real terms (divided by $P_{H,t}$), the Balance of Payments for country H can be written as:

$$\widetilde{BP}_t \equiv \widetilde{NX}_t + i_{t-1} \frac{\widetilde{NFA}_{t-1}}{\Pi_{H,t}} \quad (2.5.7)$$

Combining the relationships between net foreign assets and net exports in the two countries shows that the balance of payments of the two countries are related in the following way: $BP_t + BP_t^* = 0$. In real terms: $\widetilde{BP}_t + S_t \widetilde{BP}_t^* = 0$.

From the households' budget constraint, substituting in firm profits and labour income, and substituting in the expression for transfers backed out from the government budget constraint, yields the following relationship between assets and goods for country H :

$$D_t + B_t - (D_{t-1} + B_{t-1})(1 + i_{t-1}) = B_t^G - B_{t-1}^G(1 + i_{t-1}) + P_{H,t} Y_t - P_t C_t - P_{H,t} G_t \quad (2.5.8)$$

while substituting in the definitions of net exports and net foreign assets yields:

$$NFA_t = NFA_{t-1}(1 + i_{t-1}) + NX_t \quad (2.5.9)$$

while substituting in the definition of the balance of payments yields a relationship between net foreign assets and the balance of payments:

$$NFA_t = NFA_{t-1} + BP_t \quad (2.5.10)$$

which shows that the balance of payments is equal to the variation of net foreign assets over one period.

In real terms (divided by $P_{H,t}$), the previous equation can be rewritten as:

$$\widetilde{NFA}_t = (1 + i_{t-1}) \frac{\widetilde{NFA}_{t-1}}{\Pi_{H,t}} + \widetilde{NX}_t = \frac{\widetilde{NFA}_{t-1}}{\Pi_{H,t}} + \widetilde{BP}_t \quad (2.5.11)$$

2.6 Central Bank and Monetary Policy

The only central bank in the currency union sets monetary policy by choosing the nominal interest rate to target *union-wide* inflation through a Taylor rule.

Monetary policy follows an Inflation Targeting regime of the kind:

$$\beta(1 + i_t) = \left(\frac{\Pi_t^U}{\Pi^U} \right)^{\phi_\pi(1-\rho_i)} [\beta(1 + i_{t-1})]^{\rho_i} \quad (2.6.1)$$

where union-wide output is defined as the population-weighted geometric average of per-capita output in the two countries:

$$Y_t^U \equiv \left(\frac{Y_t}{h} \right)^h \left(\frac{Y_t^*}{1-h} \right)^{1-h} \quad (2.6.2)$$

while the union-wide CPI is analogously defined:

$$P_t^U \equiv (P_t)^h (P_t^*)^{1-h} \quad (2.6.3)$$

so that consequently union-wide inflation is defined as the population-weighted geometric average of the CPI inflations in the two countries:

$$\Pi_t^U \equiv (\Pi_t)^h (\Pi_t^*)^{1-h} \quad (2.6.4)$$

while variables without subscripts t denote their respective steady-state levels, ϕ_π represents the responsiveness of the interest rate to inflation and ρ_i is a measure of the persistence of the interest rate over time (interest rate smoothing).

2.7 Government and Fiscal Policy in a Pure Currency Union

In a Pure Currency Union each government chooses the amount of government consumption and transfers, financed by marginal tax rates on wage income and firm sales and by short-term government bonds.

In country H the government finances a stream of public consumption G_t and transfers T_t subject to the following sequence of budget constraints:

$$\int_0^h P_{H,t}(j)G_t(j) dj + \int_0^h T_t^i di + B_{t-1}^G(1 + i_{t-1}) = B_t^G + \tau_t^s P_{H,t}Y_t + \int_0^h \tau_t^w W_t N_t^i di \quad (2.7.1)$$

where the right hand side represents government income from taxation and newly issued government bonds, while the left hand side represents total government spending on consumption and transfers, and on government bonds due at the end of period t , including interest. B_t^G are government bonds issued by country H in period t , while all other variables are as explained above. Government consumption, G_t , is given by the following CES function, just like equation 2.4.16 for the demand function for firms, where we assume that the government purchases only goods produced domestically (complete home bias):

$$G_t \equiv \left(\left(\frac{1}{h} \right)^{\frac{1}{\varepsilon}} \int_0^h G_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad (2.7.2)$$

The optimal allocation of any given public expenditure within each category of goods yields the following demand function:

$$G_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\varepsilon} \frac{G_t}{h} \quad (2.7.3)$$

and total government expenditure on public consumption simplifies to:

$$\int_0^h P_{H,t}(j)G_t(j) dj = P_{H,t}G_t \quad (2.7.4)$$

The government budget constraint can be rewritten in aggregate form as:

$$P_{H,t}G_t + T_t + B_{t-1}^G(1 + i_{t-1}) = B_t^G + \tau_t^s P_{H,t}Y_t + \tau_t^w W_t N_t \quad (2.7.5)$$

while dividing the government budget constraint by $P_{H,t}$ yields the government budget constraint in real terms:

$$G_t + \tilde{T}_t + i_{t-1} \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} = \tau_t^s Y_t + \tau_t^w MC_t d_t Y_t + \tilde{B}_t^G - \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \quad (2.7.6)$$

where variables with a tilde ($\sim$) are in real terms (divided by $P_{H,t}$), and where the left hand side represents

current government expenditure and interest payments on outstanding debt, while the right hand side represents government financing of that expenditure through taxes and the possible variation of government debt.

In the consumption scenario, fiscal policy in country H chooses government consumption to stabilize the output gap counter-cyclically, while following in part an exogenous process, through the fiscal rule:

$$\frac{G_t}{G} = \left(\frac{Y_t}{Y} \right)^{-\psi_y(1-\rho_g)} \left(\frac{G_{t-1}}{G} \right)^{\rho_g} e^{\varepsilon_t} \quad (2.7.7)$$

while keeping real transfers constant and balancing the budget:

$$\tilde{T}_t = \tilde{T} \quad \tilde{B}_t^G = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \quad (2.7.8)$$

which means that fiscal policy is financed by the variation of the tax rates on wage income and firm sales from their steady state levels respectively by a share $\gamma \in [0, 1]$ and $1 - \gamma$ through the following tax rule¹³:

$$\gamma(\tau_t^s - \tau^s) = (1 - \gamma)(\tau_t^w - \tau^w) \quad (2.7.9)$$

where $\rho_g \in [0, 1]$ is a measure of persistence of the government consumption shock in its AR(1) process in logs and ε_t is a zero mean white noise process. Variables without subscripts t represent their respective steady-state level, while $\psi_y \geq 0$ represents the responsiveness of government consumption to variations of the output gap.

In the transfer scenario, fiscal policy in country H chooses real transfers to stabilize the output gap counter-cyclically, while following in part an exogenous process, through the fiscal rule:

$$\frac{\tilde{T}_t}{\tilde{T}} = \left(\frac{Y_t}{Y} \right)^{-\psi_y(1-\rho_t)} \left(\frac{\tilde{T}_{t-1}}{\tilde{T}} \right)^{\rho_t} e^{\varepsilon_t} \quad (2.7.10)$$

while keeping government consumption constant and balancing the budget:

$$G_t = G \quad \tilde{B}_t^G = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \quad (2.7.11)$$

¹³If the overall tax rate is defined as:

$$\tau_t^o \equiv \tau_t^s + \tau_t^w$$

then the variation of the tax rates on wage income and on firm sales will be given respectively by a share $\gamma \in [0, 1]$ and $1 - \gamma$ of the variation of the overall tax rate in the following way:

$$(\tau_t^w - \tau^w) \equiv \gamma(\tau_t^o - \tau^o)$$

$$(\tau_t^s - \tau^s) \equiv (1 - \gamma)(\tau_t^o - \tau^o)$$

which implies the tax rule in the text.

which means that fiscal policy is financed by the variation of the tax rates on wage income and firm sales from their steady state levels respectively by a share $\gamma \in [0, 1]$ and $1 - \gamma$ through the following tax rule:

$$\gamma(\tau_t^s - \tau^s) = (1 - \gamma)(\tau_t^w - \tau^w) \quad (2.7.12)$$

where $\rho_t \in [0, 1]$ is a measure of persistence of the transfer shock in its AR(1) process in logs and ε_t is a zero mean white noise process, while $\psi_y \geq 0$ represents the responsiveness of real transfers to variations of the output gap.

In country F the government finances a stream of public consumption G_t^* and transfers T_t^* subject to the following sequence of budget constraints:

$$\int_h^1 P_{H,t}^*(j) G_t^*(j) dj + \int_h^1 T_t^{*i} di + B_{t-1}^{*G} (1 + i_{t-1}) = B_t^{*G} + \tau_t^{*s} P_{H,t}^* Y_t^* + \int_h^1 \tau_t^{*w} W_t^* N_t^{*i} di \quad (2.7.13)$$

where the right hand side represents government income from taxation and newly issued government bonds, while the left hand side represents total government spending on consumption and transfers, and on government bonds due at the end of period t , including interest. B_t^{*G} are government bonds issued by country F in period t , while all other variables are as explained above. Government consumption, G_t^* , is given by the following CES function, just like equation 2.4.19 for the demand function for firms, where we assume that the government purchases only goods produced domestically (complete home bias):

$$G_t^* \equiv \left(\left(\frac{1}{1-h} \right)^{\frac{1}{\varepsilon}} \int_h^1 G_t^*(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad (2.7.14)$$

The optimal allocation of any given public expenditure within each category of goods yields the following demand function:

$$G_t^*(j) = \left(\frac{P_{H,t}^*(j)}{P_{H,t}^*} \right)^{-\varepsilon} \frac{G_t^*}{1-h} \quad (2.7.15)$$

and total government expenditures on public consumption simplify to:

$$\int_h^1 P_{H,t}^*(j) G_t^*(j) dj = P_{H,t}^* G_t^* \quad (2.7.16)$$

The government budget constraint can be rewritten in aggregate form as:

$$P_{H,t}^* G_t^* + T_t^* + B_{t-1}^{*G} (1 + i_{t-1}) = B_t^{*G} + \tau_t^{*s} P_{H,t}^* Y_t^* + \tau_t^{*w} W_t^* N_t^* \quad (2.7.17)$$

while dividing the government budget constraint by $P_{H,t}^*$ yields the government budget constraint in real terms:

$$G_t^* + \tilde{T}_t^* + i_{t-1} \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} = \tau_t^{*s} Y_t^* + \tau_t^{*w} MC_t^* d_t^* Y_t^* + \tilde{B}_t^{*G} - \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} \quad (2.7.18)$$

where variables with a tilde ($\sim$) are in real terms (divided by $P_{H,t}^*$), and where the left hand side represents current government expenditure and interest payments on outstanding debt, while the right hand side represents government financing of that expenditure through taxes and the possible variation of government debt.

In the consumption scenario, fiscal policy in country F chooses government consumption to stabilize the output gap counter-cyclically, while following in part an exogenous process, through the fiscal rule:

$$\frac{G_t^*}{G^*} = \left(\frac{Y_t^*}{Y^*} \right)^{-\psi_y^*(1-\rho_g^*)} \left(\frac{G_{t-1}^*}{G^*} \right)^{\rho_g^*} e^{\varepsilon_t} \quad (2.7.19)$$

while keeping real transfers constant and balancing the budget:

$$\tilde{T}_t^* = \tilde{T}^* \quad \tilde{B}_t^{*G} = \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} \quad (2.7.20)$$

which means that fiscal policy is financed by the variation of the tax rates on wage income and firm sales from their steady state levels respectively by a share $\gamma^* \in [0, 1]$ and $1 - \gamma^*$ through the following tax rule¹⁴:

$$\gamma^*(\tau_t^{*s} - \tau^{*s}) = (1 - \gamma^*)(\tau_t^{*w} - \tau^{*w}) \quad (2.7.21)$$

where $\rho_g^* \in [0, 1]$ is a measure of persistence of the government consumption shock in its AR(1) process in logs and ε_t is a zero mean white noise process. Variables without subscripts t represent their respective steady-state level, while $\psi_y^* \geq 0$ represents the responsiveness of government consumption to variations of the output gap.

In the transfer scenario, fiscal policy in country F chooses real transfers to stabilize the output gap

¹⁴If the overall tax rate is defined as:

$$\tau^{*o} \equiv \tau_t^{*s} + \tau_t^{*w}$$

then the variation of the tax rates on wage income and on firm sales will be given respectively by a share $\gamma^* \in [0, 1]$ and $1 - \gamma^*$ of the variation of the overall tax rate in the following way:

$$\begin{aligned} (\tau_t^{*w} - \tau^{*w}) &\equiv \gamma^*(\tau_t^{*o} - \tau^{*o}) \\ (\tau_t^{*s} - \tau^{*s}) &\equiv (1 - \gamma^*)(\tau_t^{*o} - \tau^{*o}) \end{aligned}$$

which implies the tax rule in the text.

counter-cyclically, while following in part an exogenous process, through the fiscal rule:

$$\frac{\tilde{T}_t^*}{\tilde{T}^*} = \left(\frac{Y_t^*}{Y^*} \right)^{-\psi_y^*(1-\rho_t^*)} \left(\frac{\tilde{T}_{t-1}^*}{\tilde{T}^*} \right)^{\rho_t^*} e^{\varepsilon_t} \quad (2.7.22)$$

while keeping government consumption constant and balancing the budget:

$$G_t^* = G^* \quad \tilde{B}_t^{*G} = \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} \quad (2.7.23)$$

which means that fiscal policy is financed by the variation of the tax rates on firm sales and wage income from their steady state levels respectively by a share $\gamma^* \in [0, 1]$ and $1 - \gamma^*$ through the following tax rule:

$$\gamma^*(\tau_t^{*s} - \tau^{*s}) = (1 - \gamma^*)(\tau_t^{*w} - \tau^{*w}) \quad (2.7.24)$$

where $\rho_t^* \in [0, 1]$ is a measure of persistence of the transfer shock in its AR(1) process in logs and ε_t is a zero mean white noise process, while $\psi_y^* \geq 0$ represents the responsiveness of real transfers to variations of the output gap.

Since government bonds are traded freely within and across borders without frictions and are perfectly substitutable because they offer the same return, the total amount of bonds held by households in both countries must equal the total amount of bonds issued by the two countries' governments, so the market clearing condition for these assets in every period t is given by:

$$B_t + B_t^* = B_t^G + B_t^{*G} \quad (2.7.25)$$

which can be rewritten in real terms (dividing by $P_{H,t}$) as:

$$\tilde{B}_t + S_t \tilde{B}_t^* = \tilde{B}_t^G + S_t \tilde{B}_t^{*G} \quad (2.7.26)$$

2.8 Government and Fiscal Policy in a Coordinated Currency Union

If the Governments of the two countries choose to coordinate, they will use their fiscal instruments to target a common objective, while maintaining independent budget constraints. Instead of using government consumption or transfers to stabilize the domestic output gap counter-cyclically, they will use the same fiscal instruments to stabilize net exports pro-cyclically. This represents the act of coordinating their policies on a common objective, which depends on the interactions between the two economies. The budget constraints of the two fiscal authorities instead remain unmodified.

In the consumption scenario, fiscal policy in country H chooses government consumption to stabilize its real net exports gap pro-cyclically, while following in part an exogenous process, through the fiscal rule:

$$\frac{G_t}{G} = \left(\frac{\widetilde{NX}_t}{\widetilde{NX}} \right)^{\psi_{nx}(1-\rho_g)} \left(\frac{G_{t-1}}{G} \right)^{\rho_g} e^{\varepsilon_t} \quad (2.8.1)$$

while keeping real transfers constant and balancing the budget:

$$\tilde{T}_t = \tilde{T} \quad \tilde{B}_t^G = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \quad (2.8.2)$$

which means that fiscal policy is financed by the variation of the tax rates on wage income and firm sales from their steady state levels respectively by a share $\gamma \in [0, 1]$ and $1 - \gamma$ through the following tax rule:

$$\gamma(\tau_t^s - \tau^s) = (1 - \gamma)(\tau_t^w - \tau^w) \quad (2.8.3)$$

where $\rho_g \in [0, 1]$ is a measure of persistence of the government consumption shock in its AR(1) process in logs and ε_t is a zero mean white noise process, while $\psi_{nx} \geq 0$ represents the responsiveness of government consumption to variations of the real net exports gap.

In the same consumption scenario, fiscal policy in country F chooses government consumption to stabilize its real net exports gap pro-cyclically, while following in part an exogenous process, through the fiscal rule:

$$\frac{G_t^*}{G^*} = \left(\frac{\widetilde{NX}_t^*}{\widetilde{NX}^*} \right)^{\psi_{nx}^*(1-\rho_g^*)} \left(\frac{G_{t-1}^*}{G^*} \right)^{\rho_g^*} e^{\varepsilon_t} = \left(\frac{S \widetilde{NX}_t}{S_t \widetilde{NX}} \right)^{-\psi_{nx}^*(1-\rho_g^*)} \left(\frac{G_{t-1}^*}{G^*} \right)^{\rho_g^*} e^{\varepsilon_t} \quad (2.8.4)$$

while keeping real transfers constant and balancing the budget:

$$\tilde{T}_t^* = \tilde{T}^* \quad \tilde{B}_t^{*G} = \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} \quad (2.8.5)$$

which means that fiscal policy is financed by the variation of the tax rates on wage income and firm sales from their steady state levels respectively by a share $\gamma^* \in [0, 1]$ and $1 - \gamma^*$ through the following tax rule:

$$\gamma^*(\tau_t^{*s} - \tau^{*s}) = (1 - \gamma^*)(\tau_t^{*w} - \tau^{*w}) \quad (2.8.6)$$

where $\rho_g^* \in [0, 1]$ is a measure of persistence of the government consumption shock in its AR(1) process in logs and ε_t is a zero mean white noise process, while $\psi_{nx}^* \geq 0$ represents the responsiveness of government consumption to variations of the real net exports gap.

In the transfer scenario, fiscal policy in country H chooses real transfers to stabilize its real net exports gap pro-cyclically, while following in part an exogenous process, through the fiscal rule:

$$\frac{\tilde{T}_t}{\tilde{T}} = \left(\frac{\widetilde{NX}_t}{\widetilde{NX}} \right)^{\psi_{nx}(1-\rho_t)} \left(\frac{\tilde{T}_{t-1}}{\tilde{T}} \right)^{\rho_t} e^{\varepsilon_t} \quad (2.8.7)$$

while keeping government consumption constant and balancing the budget:

$$G_t = G \quad \tilde{B}_t^G = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \quad (2.8.8)$$

which means that fiscal policy is financed by the variation of the tax rates on wage income and firm sales from their steady state levels respectively by a share $\gamma \in [0, 1]$ and $1 - \gamma$ through the following tax rule:

$$\gamma(\tau_t^s - \tau^s) = (1 - \gamma)(\tau_t^w - \tau^w) \quad (2.8.9)$$

where $\rho_t \in [0, 1]$ is a measure of persistence of the transfer shock in its AR(1) process in logs and ε_t is a zero mean white noise process, while $\psi_{nx} \geq 0$ represents the responsiveness of real transfers to variations of the real net exports gap.

In the same transfer scenario, fiscal policy in country F chooses real transfers to stabilize its real net exports gap pro-cyclically, while following in part an exogenous process, through the fiscal rule:

$$\frac{\tilde{T}_t^*}{\tilde{T}^*} = \left(\frac{\widetilde{NX}_t^*}{\widetilde{NX}^*} \right)^{\psi_{nx}^*(1-\rho_t^*)} \left(\frac{\tilde{T}_{t-1}^*}{\tilde{T}^*} \right)^{\rho_t^*} e^{\varepsilon_t} = \left(\frac{S \widetilde{NX}_t}{S_t \widetilde{NX}} \right)^{-\psi_{nx}^*(1-\rho_t^*)} \left(\frac{\tilde{T}_{t-1}^*}{\tilde{T}^*} \right)^{\rho_t^*} e^{\varepsilon_t} \quad (2.8.10)$$

while keeping government consumption constant and balancing the budget:

$$G_t^* = G^* \quad \tilde{B}_t^{*G} = \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} \quad (2.8.11)$$

which means that fiscal policy is financed by the variation of the tax rates on firm sales and wage income from their steady state levels respectively by a share $\gamma^* \in [0, 1]$ and $1 - \gamma^*$ through the following tax rule:

$$\gamma^*(\tau_t^{*s} - \tau^{*s}) = (1 - \gamma^*)(\tau_t^{*w} - \tau^{*w}) \quad (2.8.12)$$

where $\rho_t^* \in [0, 1]$ is a measure of persistence of the transfer shock in its AR(1) process in logs and ε_t is a zero mean white noise process, while $\psi_{nx}^* \geq 0$ represents the responsiveness of real transfers to variations of the real net exports gap.

2.9 Government and Fiscal Policy in a Full Fiscal Union

If instead of considering two fiscal authorities managing fiscal policy independently, one for each country, or coordinating their policies, but with two separate budget constraints, we consider only one fiscal authority managing fiscal policy for both countries at the same time in a coordinated way and with a consolidated budget constraint, then we can think of it as an extreme case of fiscal policy coordination and call it a Full Fiscal Union.

A Full Fiscal Union uses local government spending to manage fiscal policy at the union level with a consolidated budget constraint. The Fiscal Union finances streams of local public consumption, G_t and G_t^* , and transfers, T_t and T_t^* , subject to the consolidated budget constraint of the two national fiscal authorities:

$$P_{H,t}G_t + P_{H,t}^*G_t^* + T_t + T_t^* + \mathcal{B}_{t-1}^G(1 + i_{t-1}) = \mathcal{B}_t^G + \tau_t^s P_{H,t}Y_t + \tau_t^{*s} P_{H,t}^*Y_t^* + \tau_t^w W_t N_t + \tau_t^{*w} W_t^* N_t^* \quad (2.9.1)$$

where $\mathcal{B}_t^G$ is overall nominal government debt at time t , defined by the sum of the government debts of countries H and F :

$$\mathcal{B}_t^G \equiv B_t^G + B_t^{*G} \quad (2.9.2)$$

Dividing the government budget constraint by $P_{H,t}$ yields the government budget constraint in real terms (for country H):

$$G_t + \tilde{T}_t + S_t(G_t^* + \tilde{T}_t^*) + i_{t-1} \frac{\tilde{\mathcal{B}}_{t-1}^G}{\Pi_{H,t}} = (\tau_t^s + \tau_t^w MC_t d_t)Y_t + (\tau_t^{*s} + \tau_t^{*w} MC_t^* d_t^*)S_t Y_t^* + \tilde{\mathcal{B}}_t^G - \frac{\tilde{\mathcal{B}}_{t-1}^G}{\Pi_{H,t}} \quad (2.9.3)$$

where variables with a tilde ($\tilde{\cdot}$) are in real terms (divided by $P_{H,t}$), and where the left hand side represents current government expenditure and interest payments on outstanding debt, while the right hand side represents government financing of that expenditure through taxes and the possible variation of overall government debt, which is given by:

$$\tilde{\mathcal{B}}_t^G = \tilde{B}_t^G + S_t \tilde{B}_t^{*G} \quad (2.9.4)$$

In the consumption scenario, union-wide fiscal policy chooses government consumption in each country to stabilize its real net exports gap pro-cyclically, while following in part an exogenous process, through the fiscal rules:

$$\frac{G_t}{G} = \left(\frac{\widetilde{NX}_t}{\widetilde{NX}} \right)^{\psi_{nx}(1-\rho_g)} \left(\frac{G_{t-1}}{G} \right)^{\rho_g} e^{\varepsilon_t} \quad (2.9.5)$$

$$\frac{G_t^*}{G^*} = \left(\frac{\widetilde{NX}_t^*}{\widetilde{NX}^*} \right)^{\psi_{nx}^*(1-\rho_g^*)} \left(\frac{G_{t-1}^*}{G^*} \right)^{\rho_g^*} e^{\varepsilon_t} = \left(\frac{S \widetilde{NX}_t}{S_t \widetilde{NX}} \right)^{-\psi_{nx}^*(1-\rho_g^*)} \left(\frac{G_{t-1}^*}{G^*} \right)^{\rho_g^*} e^{\varepsilon_t} \quad (2.9.6)$$

while keeping real transfers constant and balancing the overall budget:

$$\tilde{T}_t = \tilde{T} \quad \tilde{T}_t^* = \tilde{T}^* \quad \text{and} \quad \tilde{B}_t^G = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \implies \tilde{B}_t^G - \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} = S_t \left(\frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} - \tilde{B}_t^{*G} \right) \quad (2.9.7)$$

which means that fiscal policy is financed by the variation of the tax rates on wage income and firm sales from their steady state levels respectively by a share $\gamma \in [0, 1]$ and $1 - \gamma$ in each country through the following tax rule:

$$\gamma(\tau_t^s - \tau^s) = (1 - \gamma)(\tau_t^w - \tau^w) \quad (2.9.8)$$

while distributing equally among the two countries the cost of fiscal policy by varying jointly the tax rates in the following way:

$$(\tau_t^s - \tau^s) = (\tau_t^{*s} - \tau^{*s}) \quad (2.9.9)$$

$$(\tau_t^w - \tau^w) = (\tau_t^{*w} - \tau^{*w}) \quad (2.9.10)$$

where $\rho_g \in [0, 1]$ for country H and $\rho_g^* \in [0, 1]$ for country F are measures of persistence of the government consumption shocks in their AR(1) processes in logs and ε_t is a zero mean white noise process. Variables without subscripts t represent their respective steady-state level, while $\psi_{nx} \geq 0$ for country H and $\psi_{nx}^* \geq 0$ for country F represent the responsiveness of government consumption to variations of the real net exports gap.

In the transfer scenario, union-wide fiscal policy chooses real transfers in each country to stabilize its real net exports gap pro-cyclically, while following in part an exogenous process, through the fiscal rules:

$$\frac{\tilde{T}_t}{\tilde{T}} = \left(\frac{\widetilde{NX}_t}{\widetilde{NX}} \right)^{\psi_{nx}(1-\rho_t)} \left(\frac{\tilde{T}_{t-1}}{\tilde{T}} \right)^{\rho_t} e^{\varepsilon_t} \quad (2.9.11)$$

$$\frac{\tilde{T}_t^*}{\tilde{T}^*} = \left(\frac{\widetilde{NX}_t^*}{\widetilde{NX}^*} \right)^{\psi_{nx}^*(1-\rho_t^*)} \left(\frac{\tilde{T}_{t-1}^*}{\tilde{T}^*} \right)^{\rho_t^*} e^{\varepsilon_t} = \left(\frac{S \widetilde{NX}_t}{S_t \widetilde{NX}} \right)^{-\psi_{nx}^*(1-\rho_t^*)} \left(\frac{\tilde{T}_{t-1}^*}{\tilde{T}^*} \right)^{\rho_t^*} e^{\varepsilon_t} \quad (2.9.12)$$

while keeping government consumption constant and balancing the overall budget:

$$G_t = G \quad G_t^* = G^* \quad \tilde{B}_t^G = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \implies \tilde{B}_t^G + S_t \tilde{B}_t^{*G} = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} + S_t \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} \quad (2.9.13)$$

which means that fiscal policy is financed by the variation of the tax rates on wage income and firm sales from their steady state levels respectively by a share $\gamma \in [0, 1]$ and $1 - \gamma$ in each country through the following tax rule:

$$\gamma(\tau_t^s - \tau^s) = (1 - \gamma)(\tau_t^w - \tau^w) \quad (2.9.14)$$

while distributing equally among the two countries the cost of fiscal policy by varying jointly the tax rates in the following way:

$$(\tau_t^s - \tau^s) = (\tau_t^{*s} - \tau^{*s}) \quad (2.9.15)$$

$$(\tau_t^w - \tau^w) = (\tau_t^{*w} - \tau^{*w}) \quad (2.9.16)$$

where $\rho_t \in [0, 1]$ for country H and $\rho_t^* \in [0, 1]$ for country F are measures of persistence of the transfer shocks in their AR(1) processes in logs and ε_t is a zero mean white noise process. Variables without subscripts t represent their respective steady-state level, while $\psi_{nx} \geq 0$ for country H and $\psi_{nx}^* \geq 0$ for country F represent the responsiveness of real transfers to variations of the real net exports gap.

3 Calibration

The model is calibrated¹⁵ following mainly [Ferrero \(2009\)](#), so we consider the top 5 Eurozone countries, which account for more than 80% of Eurozone GDP and we divide them into the periphery (namely, France, Netherlands, Italy and Spain), country F , and the core (namely Germany), country H . The size of country H is set according to the relative GDP size to $h = 0.4$, as Germany accounts for over 35% of Eurozone GDP.

As in [Ferrero \(2009\)](#) most of the parameters governing the economies of the two countries are set symmetrically, with the exception of the degree of price rigidity, which has been set such that in country H the average duration of a price is 4 quarters while in country F it is 5 quarters. The gross markup $\frac{\varepsilon}{\varepsilon-1}$ has been set to 1.1, which implies a net markup of 10%, and the discount factor has been chosen to match a compounded annual interest rate of 2%. Table 1 collects all calibrated parameters and steady state stances.

In the calibration, we set $\eta > \frac{1}{\sigma}$ so that C_H and C_F are substitutes and hence the substitution effect of a price change dominates the income effect. In the opposite case ($\eta < \frac{1}{\sigma}$) C_H and C_F are complements and the income effect of a price change dominates the substitution effect. This implies that fiscal policy and spillovers from one country to the other have very different effects based on the two calibrations. In our analysis we focus on the case in which C_H and C_F are substitutes because we believe it is more realistic, especially for advanced economies, and more in line with the recent literature (See [Ferrero \(2009\)](#) and [Blanchard, Erceg and Lindé \(2015\)](#) for instance), but we also consider the case in which they are complements, as a sensitivity analysis for the effects of fiscal policy, as studied in [Hjortsø \(2012\)](#).

The calibration of the two countries mainly differs in the fiscal policy parameters. In particular, the government consumption-to-GDP ratios have been set respectively to 18.7% for country H and 21.9% for country F , according to the average of the last 9 years (source ECB-SDW). The marginal tax rates on wage

¹⁵The calibration is done on the steady state values described in Appendix A.2.

Table 1: Calibrated Parameters and Steady State Stances.

Parameters	Description	Country H	Country F
h	Relative size of domestic economy	.4	.6
β	Discount factor	.995	.995
ε	Elasticity of substitution of domestic goods	11	11
$\frac{\varepsilon}{\varepsilon-1}$	Gross Price Mark-Up	1.1	1.1
η	Elasticity of substitution foreign and domestic goods	[0.3, 4.5]	[0.3, 4.5]
σ	Inverse Elasticity of intertemporal substitution	3	3
φ	Inverse Frisch Elasticity of labour supply	0.5	0.5
θ	Degree of price rigidity	3/4	4/5
α	Openness of domestic economy	.52	.361
$\frac{\alpha}{h}$	Relative openness of domestic economy	1.3	.6017
$\frac{1-\alpha}{h}$	Home bias	1.2	1.065
ψ_y	Responsiveness of fiscal policy to output gap	0.045	0.001
ψ_{nx}	Responsiveness of fiscal policy to net exports gap	0.0697	0.0092
ϕ_π	Responsiveness of monetary policy to inflation	1.5	1.5
ρ_i	Interest Rate smoothing parameter	0.8	0.8
ρ_ξ	Persistence of preference shock	0.94	0.8
ρ_a	Persistence of technology shock	0.58	0.70
ρ_g	Persistence of fiscal shock	0.74	0.81
σ_ξ	Standard deviation preference shock	0.0024	0.0086
σ_a	Standard deviation technology shock	0.0087	0.0033
σ_g	Standard deviation fiscal shock	0.0056	0.0031
$corr_\xi$	Correlation preference shock	0.625	0.625
$corr_a$	Correlation technology shock	0.418	0.418
$corr_g$	Correlation fiscal shock	0	0
Steady State Ratios	Description	Country H	Country F
$(1+i)^4 - 1$	Annualized Interest Rate	2%	2%
τ^w	Tax Rate on wage income	40.6%	27.9%
τ^s	Tax Rate on firm sales	2.5%	19.5%
$\tau^w MC + \tau^s$	Tax Revenues-to-GDP	38.49%	39.92%
$\frac{G}{Y}$	Government consumption-to-GDP	18.7%	21.9%
$\frac{T}{Y}$	Real Transfers-to-GDP	18.58%	16.81%
$\frac{NX}{Y}$	Net Exports-to-GDP	1.72%	-1.14%
$\frac{C}{Y}$	Consumption-to-GDP	79.58%	79.24%
$\alpha^* \frac{C^*}{Y}$	Exports-to-GDP	43.1%	27.47%

income have been set respectively to 40.61% for country H and 27.94% for country F in accordance to the average in the last 9 years of the labour income tax wedges, excluding social security contributions made by the employer, for the median individual, as reported in [OECD \(2015\)](#). The marginal tax rate on firm sales has been set to 19.5% for country F according to the average in the last 9 years of the VAT for France, Italy, Spain and The Netherlands as reported in [Eurostat, European-Commission et al. \(2015\)](#), while it has been calibrated for country H to match the average ratio of net exports-to-GDP of 1.73% observed over the past 9 years for Germany¹⁶. Although the observed VAT rate for Germany is 19%, we set its marginal tax rate on firm sales to 2.5%, as if there was a production incentive, to correct for the fact that country H should have a greater productivity compared to country F , as Germany has a greater productivity than the periphery countries. This calibration implies a steady state tax revenue-to-GDP ratio of respectively 38.49% for country H and 39.92% for country F , clearly in line with the data observed over the past decades for Germany (38.72%) and for France, Italy, Spain and The Netherlands (39.15%). Finally, the annualized steady state value of government debt-to-GDP in both countries is set to roughly 60% as stated in the Maastricht Treaty.

Since the two countries' fiscal policy ratios have been calibrated according to the data, the transfers-to-GDP ratios have been set such that the government deficit is zero in steady state:

$$\frac{\tilde{T}}{\tilde{Y}} = (\tau^s + \tau^w MC) - \frac{G}{\tilde{Y}} - \left(\frac{1}{\beta} - 1\right) \frac{\tilde{B}^G}{\tilde{Y}} \quad (3.0.1)$$

$$\frac{\tilde{T}^*}{\tilde{Y}^*} = (\tau^{*s} + \tau^{*w} MC^*) - \frac{G^*}{\tilde{Y}^*} - \left(\frac{1}{\beta} - 1\right) \tilde{B}^{*G} \quad (3.0.2)$$

Henceforth, the overall calibration of the fiscal sector implies a steady state ratio of transfers-to-GDP of respectively 18.58% for country H and 16.81% for country F , and a steady state ratio of current expenditure-to-GDP of respectively 37.28% for country H and 38.71% for country F . This calibration is broadly in line with the observed data over the last 10 years for the subsidies-to-GDP ratio (26.85% for Germany and 24.69% for the other countries) and the current expenditure (less interest)-to-GDP ratio (35.54% for Germany and 36.85% for the other countries).

The parameters of openness have been set to match an export-to-GDP ratio $\left(\frac{\alpha^* C^*}{Y}\right)$ of roughly 43% for country H ¹⁷ taken from the aggregate demand equation, while for country F the parameter of openness is

¹⁶The average current account to GDP ratio observed over the past 9 years for Germany is roughly 6.36%. However, we adjusted the data for the overall trade weight with France, Italy, Spain and Netherlands (26%).

¹⁷The value recovered from the data as the average of the last 9 years is 43.5%.

recovered by equating per-capita consumption across countries, which yields the following equation:

$$\alpha^* = \frac{h}{1-h} \left[\alpha + \frac{\left(\frac{1-\frac{G}{Y}}{1-\frac{G^*}{Y^*}} \right) \left(\frac{(1-\tau^w)(1-\tau^s)}{(1-\tau^{*w})(1-\tau^{*s})} \right)^{\frac{1}{\varphi}} - 1}{1 + \frac{h}{1-h} \left(\frac{1-\frac{G}{Y}}{1-\frac{G^*}{Y^*}} \right) \left(\frac{(1-\tau^w)(1-\tau^s)}{(1-\tau^{*w})(1-\tau^{*s})} \right)^{\frac{1}{\varphi}}} \right] \quad (3.0.3)$$

Consequently, home biases are given by $\frac{1-\alpha}{h} = 1.2$ and $\frac{1-\alpha^*}{1-h} = 1.065$. Since both home biases are larger than one it means that the share of consumption of domestic goods is higher than the share of production of domestic goods. This generates a gap between the relative production price indices and the relative consumption price indices based on the different composition of the households' consumption basket in the two countries. Hence, the dynamics of the real exchange rate follow the dynamics of the terms of trade in a non-linear way. As Figure 1 shows, the real exchange rate increases as the terms of trade increase if the degree of openness of country H is less than the size of country F ($1-h = 0.6$), which is the case for our calibration ($\alpha = 0.52$), while the real exchange rate decreases when the terms of trade increase if the degree of openness of country H is more than the size of country F ($h = 0.4$). These two conditions imply respectively, given equation 3.0.3, that the degree of openness of country F is less than the size of country H ($h = 0.4$) for the real exchange rate to increase as the terms of trade increase, and that the degree of openness of country F is more than the size of country H ($h = 0.4$) for the real exchange rate to decrease as the terms of trade increase.

Regarding the dynamic parametrization of the shocks, all three exogenous shocks are assumed to follow a VAR(1) process that generally allows for both direct spillovers and second order correlation of the innovations. However, the structure has been restricted for both the technology shocks and the preference shocks to exclude direct spillovers and for the fiscal shocks to assume fully independent stochastic processes.

With the exception of the preference shocks, whose dynamics have been calibrated following [Kollmann et al. \(2014\)](#), the parameters characterizing the dynamics of both technology shocks and fiscal shocks have been estimated. For the estimation we have employed the time series for Germany, France, Italy and Spain of respectively final government consumption expenditures for the fiscal shocks and labour productivity per hours worked for the technology shocks. All the series are chain-linked volumes re-based respectively in 2005 (fiscal shocks) and 2010 (technology shocks), seasonally adjusted and filtered by means of a Hodrick-Prescott filter. The sample considered spans at quarterly frequency from 2002 Q1 to 2014 Q2 for the fiscal shocks and to 2015 Q3 for the technology shocks. Finally, despite a large debate on the high correlation between preference shocks in the Eurozone, there is no proper reference in the literature for its calibration. We decide to set this parameter according to the observed business cycle correlation (which is roughly 0.5) and we pick the value that maximizes the simulated correlation between output in the two countries (which is roughly

Figure 1: Elasticity of the Real Exchange Rate to the Terms of Trade

$0.42)^{18}$.

3.1 Sensitivity Analysis

Figures 2 and 3 show a sensitivity analysis for the tax rates on wage income and firm sales in steady state for each of the two countries, where the tax rates on wage income and firm sales that maximize overall tax revenues in each country are shown in red, while the calibrated tax rates on wage income and firm sales are shown in green.

We can see from the graphs that the revenue maximizing value for the tax rate on wage income is zero because it is highly distortionary, which implies that the distortion it creates through a reduction in the tax base dominates the increase in the tax revenues. On the other hand the revenue maximizing value for the tax rate on firm sales is between 40% and 50%, which implies it is much less distortionary compared to the

¹⁸The simulated values of the correlation of business cycles in our model, given our calibration, are always lower than the observed correlation. Therefore, we decide to select the correlation of preference shocks that maximizes the correlation of business cycles.

Figure 2: Laffer Curve for Country H

tax rate on wage income, so the burden of adjustment should be mainly borne by the tax rate on firm sales. Notice also that if the tax rate on firm sales exceeds the values in red, then it will reduce tax revenues and might as well have the opposite effects on the economy.

These graphs also show how relatively far the calibrations of the tax rates are from their revenue maximizing values, so that for the given calibration revenues are increasing in the tax rate on firm sales and decreasing in the tax rate on wage income. At the same time the revenue maximizing values are close for the two countries, because the dynamics are similar, while the calibrations are quite different because of the different structure of the two economies.

4 Welfare Analysis and Optimal Fiscal Policy Parameters

The selection of the optimal fiscal policy parameters follows from the analysis of the fiscal policy rules used in our model. In the search for the optimal fiscal policy parameters, we limit the analysis to an expenditure rule that sets government consumption to target either the output gap or the net exports gap. Specifically, these fiscal policy parameters have been selected to maximize the unconditional expectation of lifetime utility of the total population of households¹⁹ under the condition that they induce a locally unique rational expectations equilibrium²⁰. Therefore the fiscal policy rules in our model are judged optimal in their class

¹⁹Even if in the Pure Currency Union scenario the fiscal decisions are taken independently, we consider the results of the joint maximization of average aggregate welfare because it is in line with the results of a dynamic game between the two countries.

²⁰Following [Schmitt-Grohé and Uribe \(2007\)](#), we discretized the policy space by means of a grid search, because welfare is a non monotonic function of the fiscal policy parameters and has several local maxima. We consider 100 different values for

Figure 3: Laffer Curve for Country F

of rules, because the fiscal policy parameters are chosen to yield the highest average level of welfare to the representative household compared to all other fiscal policy parameters.

As a measure of welfare we consider the weighted average of the second order approximation of the utility of households in each country, given by:

$$\widetilde{W}_t = hW_t + (1 - h)W_t^* \quad (4.0.1)$$

where welfare for country H is given by:

$$W_t = \xi_t \left(\frac{\left(\frac{C_t}{h}\right)^{1-\sigma} - 1}{1 - \sigma} - \frac{\left(\frac{Y_t d_t}{A_t h}\right)^{1+\varphi}}{1 + \varphi} \right) + \beta W_{t+1} \quad (4.0.2)$$

and welfare for country F is given by:

$$W_t^* = \xi_t^* \left(\frac{\left(\frac{C_t^*}{1-h}\right)^{1-\sigma} - 1}{1 - \sigma} - \frac{\left(\frac{Y_t^* d_t^*}{A_t^* (1-h)}\right)^{1+\varphi}}{1 + \varphi} \right) + \beta W_{t+1}^* \quad (4.0.3)$$

Although we select the fiscal policy parameters based on the unconditional expectation of lifetime utility, each target variable (i.e. either the output gap or the net exports gap) and limit the parameter space to lie between 0 and 0.1 or between -0.1 and 0 based on the expected sign of the parameter, because a larger parameter space would imply a non stationary equilibrium given by the distortionary effect of taxation overcoming the stabilizing effect of government spending.

to compare welfare attained under alternative fiscal policy scenarios we prefer to rely on the expectation of lifetime utility conditional on the initial state being the non-stochastic steady state. In this way, the welfare ranking of alternative policies will depend on the assumed value and distribution of the initial state vector (x_0). This measure accounts for the transitional dynamics leading back to the stochastic steady state and, as the deterministic steady state is the same across all the scenarios considered, we ensure that the economy begins from the same initial point under all possible policies.

4.1 Welfare Gains based on Consumption Equivalent Variations

Following [Schmitt-Grohé and Uribe \(2007\)](#), we compute the welfare gain of a particular fiscal policy scenario relative to our benchmark scenario: the Pure Currency Union scenario with exogenous government consumption. We denote the benchmark policy scenario with b , the alternative scenarios with a , and the steady state scenario with 0, and we consider the welfare gain λ as the percentage increase in the benchmark scenario's expected consumption that leaves the representative household as well off as in the alternative scenario. Therefore λ can be recovered from the following identity:

$$E\{W_a\} = \frac{\xi_b}{(1-\beta)} \left(\frac{\left(\frac{(1+\lambda)C_b}{h} \right)^{1-\sigma} - 1}{1-\sigma} - \frac{\left(\frac{Y_b d_b}{A_b h} \right)^{1+\varphi}}{1+\varphi} \right) =$$

$$(E\{W_b\} - W_0)(1+\lambda)^{(1-\sigma)} + \frac{(1+\lambda)^{(1-\sigma)} - 1}{(1-\sigma)(1-\beta)} + W_0 \quad (4.1.1)$$

The welfare gain is then equal to:

$$\lambda = \left[\frac{(1-\sigma)(E\{W_a\} - W_0) + (1-\beta)^{-1}}{(1-\sigma)(E\{W_b\} - W_0) + (1-\beta)^{-1}} \right]^{\frac{1}{1-\sigma}} - 1 \quad (4.1.2)$$

Note that in the equation above λ is a function of both the initial conditions (x_0) and the expected variance (σ_0), because it is a function of the conditional expectations of welfare, which in turn depend on x_0 and σ_0 . To compute the value of λ we consider its Taylor expansion around the point $x = x_0$ and $\sigma_0 = 0$. Since we choose the initial state to be the deterministic steady state, we need to consider a second-order approximation of λ because only the second derivatives of welfare with respect to σ are non-zero. Indeed, since the steady state is the same across all scenarios, λ vanishes around the point (x_0, σ_0) and the first derivatives with respect to σ are null. Totally differentiating twice the welfare gain λ and evaluating the results at (x_0, σ_0) yields:

$$\lambda \approx \left(\frac{\partial^2 W_b}{\partial \sigma^2} - \frac{\partial^2 W_a}{\partial \sigma^2} \right) (1-\beta) \quad (4.1.3)$$

The optimal fiscal policy parameters and the welfare gains based on Consumption Equivalent Variations are reported in Table 2. The optimal coefficients have been selected respectively under the Pure Currency Union scenario for the response of government consumption to the output gap and under the Coordinated Currency Union scenario for the response of government consumption to the net exports gap²¹.

Table 2: Optimal Fiscal Policy Parameters and Welfare Gains based on CEV

Policy Scenarios	Optimal Parameters*		Conditional Welfare Gains		
	ψ	ψ^*	Country H	Country F	Average
PCU	0	0	0%	0%	0%
PCU	0.045	0.001	0%	0.86%	0.51%
CCU	0.0697	0.0092	2.17%	0.02%	0.88%
FFU	0.0697	0.0092	0.03%	0.27%	0.18%
FFU	0	0	0.03%	-0.09%	-0.04%

*The optimal parameters are selected looking at the maximum unconditional expected lifetime utility.

From Table 2 we can see that the average welfare gain is highest in the Coordinated Currency Union scenario compared to other scenarios, because stabilizing net exports is found to be welfare improving compared to stabilizing output. Second place for average welfare is held by the Pure Currency Union scenario, which has a greater welfare gain compared to the the Full Fiscal Union scenario, mainly because the latter compared to the former reduces a lot welfare in country H and increases a little welfare in country F , because of the distributional effects created by the consolidation of budget constraints.

The highest welfare gain for country H is in the Coordinated Currency Union scenario, because it is the country with positive net exports and thus has more to gain in stabilizing the net exports gap. Also, since country H has a lower degree of price rigidity, after a shock prices move more than in country F , so that in country H the direct effect on output (income effect) and the indirect effect through the terms of trade (substitution effect) move in the same direction, bringing the economy further away from the initial equilibrium. Thus, stabilizing net exports yields greater welfare gains because it counteracts the substitution effect, reducing the negative effects of the shock. Note that, although small, there is a welfare gain for country H in the Full Fiscal Union scenario compared to the Pure Currency Union scenario, given by the distributional effects of the consolidation of budget constraints, that puts more burden of financing on the country with higher output (country F). At the same time Table 2 shows that targeting output rather than having stochastic government consumption produces no welfare gains for country H .

The highest welfare gain for country F is instead in the Pure Currency Union scenario, because it is

²¹The welfare gains for government transfers as a policy instrument are not reported, and the selected coefficients for government transfers are set equal to the optimal coefficients for government consumption, to make them comparable and to focus more on government consumption as a policy instrument.

the country with negative net exports and thus has more to gain in stabilizing the output gap. Also, since country F has a higher degree of price rigidity, after a shock prices move less than in country H , so that in country F the income effect and the substitution effect move in opposite directions not allowing the economy to go far away from the initial equilibrium, and thus stabilizing output yields greater welfare gains because it allows country F to partly offset the higher degree of price rigidity by letting the terms of trade and thus net exports fluctuate freely. Note that, although small, there is a welfare gain for country F in the Coordinated Currency Union scenario and the Full Fiscal Union scenario, but only compared to the Pure Currency Union scenario with stochastic government consumption. At the same time, the Full Fiscal Union scenario with stochastic government consumption has a welfare cost for country F , which is given by the additional burden of financing fiscal policy given by the distributional effect of consolidating budget constraints, since country F has a larger output.

We can also see from Table 2 that using a targeting rule, whichever the target, rather than having exogenous government consumption, is always welfare improving for any country because rules are meant to stabilize a variable and thus reduce its volatility, which yields always a welfare gain. We can also notice that the scenario which minimizes the difference in welfare gains between countries is the Full Fiscal Union scenario, which is in fact meant to treat both countries as equally as possible. In the Full Fiscal Union scenario the welfare gains for country H from targeting the net exports gap are coupled with the distributional effects of consolidated budget constraints, which reduce the welfare gains for country H increasing the welfare gains for country F . Although country F has practically no welfare gains from targeting the net exports gap or consolidating budget constraints separately, it attains quite a welfare gain from the two together, showing that the consolidation of budget constraints has a distributional effect on welfare gains too, so that country F not only bears a higher financing burden, but also attains a higher welfare gain from consolidation.

4.2 Welfare Gains based on an ad hoc Loss Function

Blanchard, Erceg and Lindé (2015) argues that utility-based welfare measures probably underestimate the benefits of reducing the output gap in economies facing a high resource slack (negative net exports), as in the Euro Area periphery. Explicitly, the utility-based welfare measure shows less benefits from fiscal expansions than a simple ad hoc welfare measure because net exports play a substantial role in reducing the periphery's output gap and the increase in consumption in the periphery is delayed so that it has very small welfare effects.

We decide to compare the policy scenarios also based on an ad hoc loss function for the reasons stated above, as in Blanchard, Erceg and Lindé (2015). Since fiscal policy has a stabilizing function, it mimics

the behavior of monetary policy, and together they reduce both the inflation gap and the output gap. Furthermore, there are gains in terms of consumption and unemployment related to closing the output gap that are underestimated by utility-based measures. Hence, the gains from fiscal spillovers between the core and the periphery lie between the ones based on consumption equivalent variations and the ones based on an ad hoc loss function.

Using a standard quadratic loss function, the policymakers are assumed to care only about minimizing the square of the output gap and of the inflation gap in both regions. Each region's loss function is, hence, simply the sum of the square of the inflation gap and the square of the output gap, with weights 3 and 1 respectively. The overall loss function is the weighted average of each region's loss function:

$$Loss = \sum_{j=0}^{\infty} \beta^j \left\{ h \left[(\hat{\pi}_{t+j})^2 + \frac{1}{3} (\hat{Y}_{t+j})^2 \right] + (1-h) \left[(\hat{\pi}_{t+j}^*)^2 + \frac{1}{3} (\hat{Y}_{t+j}^*)^2 \right] \right\} \quad (4.2.1)$$

Table 3: Welfare Gains based on ad hoc loss function

Policy Scenario	Losses			Welfare Gains*		
	Country H	Country F	Average	Country H	Country F	Average
PCU ($\psi = \psi^* = 0$)	0.6226	0.1899	0.1388	0	0	0
PCU	0.8872	0.2696	0.1972	-42.51%	-41.92%	-42.02%
CCU	0.0416	0.0969	0.0748	93.31%	94.90%	94.61%
FFU	0.0351	0.0744	0.0587	94.35%	96.08%	95.77%
FFU ($\psi = \psi^* = 0$)	0.0409	0.0906	0.0707	93.42%	95.23%	94.91%

Welfare Gains are computed as $\frac{Loss_b - Loss_a}{Loss_b}$, with $Loss_b$ the loss of the PCU with $\psi = \psi^ = 0$.

From Table 3 we can see that the average welfare gain is greater in the Full Fiscal Union scenario compared to other scenarios, because targeting the net exports gap reduces the overall inflation gap and consolidating budget constraints reduces the output gap, providing overall stabilization for both countries. Second place for average welfare is held by the Coordinated Currency Union scenario, which has welfare gains compared to the Pure Currency Union scenario which has welfare losses, mainly because targeting the net exports gap reduces international spillovers by stabilizing overall output and inflation, with little difference in welfare gains compared to the Full Fiscal Union scenario. At the same time we can see that targeting the output gap is welfare reducing, compared to stochastic government consumption, which means that output is stabilized more by targeting net exports or nothing.

We can see from Table 3 that the welfare gains for both countries, both individually and on average, are increasing in the degree of coordination. The fact that both countries incur in big welfare losses in the Pure Currency Union scenario, while incurring in welfare gains in the other scenarios, shows the big welfare gains

from either targeting the net exports gap compared to the output gap or consolidating budget constraints. As a matter of fact, adding one dimension to the other yields very small additional welfare gains, as most of the gains take place by *either* targeting net exports *or* consolidating budget constraints. What is quite surprising is that, according to this welfare measure, even only consolidating budget constraints yields welfare gains similar and a little greater to those reached by targeting the net exports gap compared to stochastic government consumption. This is because a consolidated budget constraint stabilizes the inflation gap and the output gap on its own somehow, just like targeting the net exports gap does.

5 Numerical Simulations

We simulate the model numerically using Dynare²², which takes a second-order approximation of the model around its symmetric non-stochastic steady-state with zero inflation and constant government debt. We compare the Impulse Response Functions of the main variables to negative shocks of one standard deviation of different nature, under a range of fiscal policy specifications, to study the stabilization properties of different fiscal policy instruments, financing schemes and coordination strategies and to study the international transmission of fiscal policy shocks.

In the following graphs we compare the impulse responses to a negative technology shock in country H and the impulse responses to a negative preference shock in country F . These two shocks represent better than other shocks the relevant dynamics in the Eurozone, and give the most amplified dynamics. This is because a supply shock is more relevant in a country like Germany (country H), which is a main producer and exporter of goods and services in the EMU, while a demand shock is more relevant for periphery countries (country F), which are mainly consumers and importers in the EMU. We also compare impulse responses to fiscal shocks in both countries.

5.1 Fiscal Policy Instruments

In the first set of graphs we compare different fiscal policy instruments in both the Pure Currency Union scenario and the Full Fiscal Union scenario: countercyclical government consumption and countercyclical real transfers with respect to stochastic government consumption.

Figure 4 shows the impulse responses with different fiscal instruments to a negative preference shock in country F under the Pure Currency Union scenario. Because of the assumption of complete markets, domestic and foreign consumption move in a similar way, and they reduce on impact after a negative preference shock. As a consequence real GDP goes down, while inflation diminishes bringing to an increase

²²All the equilibrium conditions of the model used for the simulations are shown in Appendix A.1.

Figure 4: Fiscal Instruments - Pure Currency Union - Preference Shock in Country F

in the terms of trade because of the difference in price rigidity, inducing a variation of net exports. The reduction in inflation in both countries induces the common central bank to lower policy rates, while the response of national fiscal authorities varies according to the regime. Countercyclical fiscal policy implies an increase of government consumption or transfers, which calls for higher taxation to keep the budget balanced. Therefore, private consumption and output are below their steady state values for about ten quarters, with a more severe recession in country F because net exports increase in country H (which means they decrease in country F). Interestingly, most variables' responses still follow a similar path when fiscal policy uses countercyclical government consumption as when it uses stochastic government consumption, with the latter creating slightly more stabilized dynamics. Using countercyclical government transfers though, after a preference shock, brings to more amplified dynamics for all variables.

After a technology shock²³, which reduces labour supply and income, using countercyclical transfers

²³The model has been simulated after both a technology and a preference shock, but for synthesis we report and discuss only the most relevant cases. The graphs related to the other shocks are available upon request.

Figure 5: Fiscal Instruments - Full Fiscal Union - Preference Shock in Country F

reduces the effects of the shock on prices and thus the interest rate, because it affects consumption which is aimed at both domestic and foreign goods, while using countercyclical government consumption affects prices more and directly, because it is aimed only at domestic goods. Instead, after a preference shock which reduces consumption and thus prices and the interest rate, government transfers are not able to counteract the decrease in consumption and the fall in prices right away, but only partially and with a delay, compared to government consumption which, since it is only aimed at domestic goods, counteracts the fall in prices and thus the fall in the interest rate, stabilizing consumption more.

This implies that the best instrument for fiscal policy in a Pure Currency Union scenario depends on the type of shock: government consumption is better in stabilizing consumption after a preference shock, while transfers are slightly better in stabilizing consumption after a technology shock. To abstract from the nature of the shock, we use countercyclical government consumption as an instrument for most simulations, because it is generally less volatile than using government transfers.

Figure 5 shows the impulse responses with different fiscal instruments to the same negative preference

shock in country F of figure 4, under the Full Fiscal Union scenario. The qualitative response of the variables after the shock is similar to the Pure Currency Union scenario, but now the dynamics are not significantly affected by the fiscal instrument used by the government, because consolidating budget constraints provides on its own almost all the stabilization, leaving little job for the fiscal policy instrument. At this point only countercyclical government consumption can provide a little more stabilization, by targeting net exports and purchasing only domestic goods, compensating directly international demand imbalances. To achieve as much stabilization as possible the best fiscal instrument to use is countercyclical government consumption, especially in the Full Fiscal Union case, even if the gain is not that big.

5.2 Financing Fiscal Policy

Here we compare different financing schemes for fiscal policy, varying the percentage financed by the tax rate on wage income and the tax rate on firm sales. More in detail, we simulate the model under three combinations of τ^s and τ^w :

- $\gamma = 0.2$, financed roughly 20% by varying the tax rate on wage income and 80% by varying the tax rate on firm sales.
- $\gamma = 0.5$, financed roughly by varying equally the two tax rates. This can be considered as the baseline financing scheme, followed in all other sections.
- $\gamma = 0.8$, financed roughly 80% by varying the tax rate on wage income and 20% by varying the tax rate on firm sales.

We also compare the outcomes of financing fiscal policy with lump-sum taxes, which do not produce distortions in the economy. Figure 6 shows the impulse responses with different financing schemes to a negative technology shock in country H under the Pure Currency Union scenario²⁴

After a technology shock in country H , most variables' responses are quite more amplified when fiscal policy is financed by varying more the tax rate on wage income ($\gamma = 0.8$), because it is more distortionary than the tax rate on firm sales. The most stabilized dynamics are given by varying mainly the tax rate on firm sales ($\gamma = 0.2$), which is less distortionary than the tax rate on wage income. At the same time, varying equally the two tax rates ($\gamma = 0.5$) creates little more distortion compared to $\gamma = 0.2$ and much less distortion compared to $\gamma = 0.8$. Thus, the amplification of the shocks is increasing exponentially in γ , starting from non-distortionary lump-sum taxes, which follow similar but much less amplified dynamics, to the most amplified dynamics given by the massive use of the tax rate on wage income to finance fiscal policy.

²⁴The results are similar after a preference shock in country F , so we don't show the impulse responses, although they are available on request.

Figure 6: Fiscal Financing - Pure Currency Union - Technology Shock in Country H

This implies that fiscal policy has a greater stabilization role when it is mainly financed with taxation on firm sales because it is less distortionary. In fact the tax rate on wage income affects very much the equilibrium level of output because of its impact on labour supply, while the tax rate on firm sales affects primarily prices and inflation, with secondary effects on output. We could reduce γ to a minimum, but we prefer to use balanced financing ($\gamma = 0.5$) in all other simulations, as it appears to be more consistent with the empirical evidence. Notice that the tax mix does not affect qualitatively the dynamics, even with lump-sum taxes (non distortionary taxation).

5.3 Fiscal Policy Coordination

In the following graphs we compare the three different degrees of fiscal policy coordination, Pure Currency Union, Coordinated Currency Union, and Full Fiscal Union, after a negative technology shock in country

Figure 7: Fiscal Policy Coordination - Technology Shock in Country H

H and after a negative preference shock in country F ²⁵. The impulse responses are shown in Figure 7 and Figure 8 respectively.

Looking at Figure 7 and 8, we can see how the dynamics are much more amplified under a Pure Currency Union scenario and much less amplified under a Coordinated Currency Union scenario. This is because when government consumption targets net exports rather than output, international spillovers are reduced and the economy is more stable. Specifically, net exports fluctuate highly after a shock, due to the re-balancing of the household consumption baskets following movements in the terms of trade. By stabilizing net exports the terms of trade are consequently more stable, reducing the substitution effect of a price change, and making consumption and output less volatile than in the Pure Currency Union scenario.

Furthermore, since government consumption is financed with distortionary tax rates, the stabilizing effect of government consumption on output is small because of the opposite effect on consumption of the

²⁵The financing scheme for these simulations is given by a balanced mix of the two tax rates, corresponding to the case $\gamma = 0.5$ analyzed in the previous section.

Figure 8: Fiscal Policy Coordination - Preference Shock in Country F

movements in the tax rates. This is especially true in the Pure Currency Union scenario, where fiscal policy cannot achieve much output stabilization. Whereas, re-balancing domestic demand to be in line with domestic supply, by stabilizing net exports, is a target that fiscal policy can achieve. The Full Fiscal Union scenario presents dynamics which are very close to those in the Coordinated Currency Union scenario, because in both cases government consumption targets net exports. As a consequence, we provide evidence of a significant gain in terms of stabilization when the government targets net exports, while if we also consolidate budget constraints we obtain very small improvements in terms of stabilization. However, government consumption reacts less in the Full Fiscal Union scenario than in the Coordinated Currency Union scenario, because consolidating budget constraints produces international stabilization on its own.

6 Fiscal Shocks and Fiscal Multipliers and Spillovers

In the previous sections we assessed the stabilizing properties of different degrees of fiscal policy coordination, but we can also evaluate the effects of a fiscal stimulus under the same scenarios. In this section we study fiscal multipliers and spillovers, which describe the effects of exogenous variations of government consumption on domestic and foreign output respectively. Although in Europe fiscal multipliers are typically smaller than in the U.S., [Erceg et al. \(2012\)](#), using several macroeconomic models, found that in the Eurozone the effects of government spending are sizable but require a large fiscal space, especially in periods of economic downturn, and monetary support (i.e. holding the interest rate constant for some periods). Furthermore, in a monetary union, a fiscal stimulus implemented in a country with larger fiscal space (such as Germany) can create positive international spillovers.

6.1 Steady State Fiscal Multipliers and Spillovers

Fiscal policy multipliers in steady state are defined as the effect of G on Y , while fiscal policy spillovers in steady state are defined as the effect of G^* on Y . Fiscal policy multipliers and spillovers in steady state in country H are given respectively by the following equations:

$$\frac{\partial Y}{\partial G} = \left[1 - \frac{(1-\alpha)h + \alpha^*(1-h)}{(1-\alpha^*)(1-h) + \alpha h} \frac{1-h}{h} \left[\frac{(1-\tau^{*s})(1-\tau^{*w})}{(1-\tau^s)(1-\tau^w)} \right]^{\frac{1}{\varphi}} \right]^{-1} \quad (6.1.1)$$

$$\frac{\partial Y}{\partial G^*} = - \left[\frac{(1-\alpha)h + \alpha^*(1-h)}{(1-\alpha^*)(1-h) + \alpha h} \right] \left[1 - \frac{(1-\alpha)h + \alpha^*(1-h)}{(1-\alpha^*)(1-h) + \alpha h} \frac{1-h}{h} \left[\frac{(1-\tau^{*s})(1-\tau^{*w})}{(1-\tau^s)(1-\tau^w)} \right]^{\frac{1}{\varphi}} \right]^{-1} \quad (6.1.2)$$

These equations show that fiscal multipliers mainly depend on the degree of openness of the two countries and, hence, international demand imbalances are a key determinant in the dynamics of output after a fiscal shock. Specifically, from the previous equations, sufficient, but not necessary, conditions for fiscal multipliers to be positive and greater than 1 and at the same time for fiscal spillovers to be negative are:

$$\frac{\alpha^*}{\alpha} < \frac{h}{1-h} \quad (6.1.3)$$

$$\tau^{*s} + \tau^{*w} - \tau^{*s}\tau^{*w} > \tau^s + \tau^w - \tau^s\tau^w \quad (6.1.4)$$

This tells us that when fiscal multipliers are positive and greater than 1, fiscal spillovers are negative, and viceversa. Given our calibration, the conditions listed above are satisfied with opposite inequality signs and, hence, as reported in [Table 4](#), in country H in steady state the fiscal multiplier is negative and the fiscal spillover is positive. This implies that in country F in steady state the fiscal multiplier is positive and the

fiscal spillover is negative. This is because the degree of openness of country H is much higher than that of country F.

Table 4: Fiscal Multiplier and Fiscal Spillover in SS

	Fiscal Multiplier	Fiscal Spillover	G/Y
Country H	-24.4062	16.8643	0.187
Country F	25.4062	-36.7682	0.219

Notice that these fiscal multipliers and spillovers do not consider the effects of how the government consumption is financed, which implies variations of the tax rates as a function of the variations of G and G^* , but they do include both supply and demand factors.

6.2 Short-Run Fiscal Multipliers and Spillovers

In this section, we compare the short-run fiscal policy multipliers and spillovers for both countries H and F in the Pure Currency Union scenario and in the Full Fiscal Union one. The short-run fiscal policy multiplier is defined as the percentage deviation of domestic output given by a 1% deviation of domestic government consumption (or transfers), while the fiscal policy spillover is defined as the percentage deviation of foreign output given by a 1% deviation of domestic government consumption (or transfers)²⁶. To study the effects of fiscal policy shocks, we show the dynamics of output and taxes after these shocks in each country. Notice that in order to compute the fiscal policy multipliers and spillovers we have to consider government consumption (or transfers) as purely stochastic, so that the only difference between the Pure Currency Union scenario and the Full Fiscal Union one is given by the consolidation of budget constraints.

Figure 9 shows fiscal policy multipliers and spillovers, output and taxes, after a positive fiscal policy shock in country H financed by a mix of taxes ($\gamma = 0.5$), comparing the Pure Currency Union scenario with the Full Fiscal Union one. Figure 10 reports the same analysis by considering a positive fiscal policy shock in country F .

The figures show that there is a relationship between fiscal policy multipliers and spillovers: the bigger the fiscal policy multiplier in one country over time, the bigger the fiscal policy spillover on the other country. The graphs also show that fiscal policy multipliers and spillovers are smaller in absolute terms in the Full Fiscal Union scenario compared to the Pure Currency Union scenario, after both a shock to government

²⁶The fiscal policy multiplier is computed as the log deviation of domestic output over the log-deviation of domestic government consumption (or transfers) after a positive one standard deviation shock to domestic government consumption (or transfers), while the fiscal policy spillover is computed as the log-deviation of foreign output to a fiscal policy shock in the home country

Figure 9: Fiscal Multipliers and Spillovers - Fiscal Shock in Country H

consumption or transfers. This is achieved through the consolidation of budget constraints, as it allows to distribute tax distortions across countries, leading in turn to more stable dynamics of taxes and output. Moreover, in the Pure Currency Union scenario, the fiscal policy multiplier and spillover change sign after about 6 quarters, given by the persistence of the fiscal policy shock. This path is driven mainly by the distortionary taxation, whose effect overturns the stimulus given by the expansionary fiscal policy.

Another important result is that fiscal multipliers and spillovers are positive after a fiscal shock in country F , while they are negative after a fiscal shock in country H . This is mainly given by the difference in price rigidity and trade openness between the two countries and, in turn, the different effects that government spending has on the terms of trade. In country H , where there is lower price rigidity, an increase in government spending brings about an increase in the terms of trade and in net exports. As a consequence, consumption and output decrease in both countries, because of complete markets. On the other hand, in country F , where prices are more rigid, an increase in government spending induces a decrease in the terms of trade and in net exports, fostering higher consumption and output in both countries, also because of complete markets. The dynamics of taxes is another key factor explaining why output in country H

Figure 10: Fiscal Multipliers and Spillovers - Fiscal Shock in Country F

increases only in the Full Fiscal Union scenario, reversing the effect of government consumption on output compared to the Pure Currency Union scenario.

All the simulations conducted suggest that generally a Full Fiscal Union produces more stabilization in the economy, through the reduction of international spillovers attained by targeting a common international variable such as net exports and/or by consolidating budget constraints, as we have seen in this section. This also implies that fiscal policy multipliers and spillovers are greatly reduced in the Full Fiscal Union scenario compared to the Pure Currency Union scenario. The stabilization is also induced by the larger fiscal capacity observed under the Full Fiscal Union scenario, which allows one country to benefit from the other country financing part of its expenditure, as if it received transfers from it or as if it were allowed to increase its government debt.

6.3 Fiscal Shocks and Fiscal Policy Coordination

Figures 11 and 12 show the impulse responses to a negative fiscal policy shock in country H and F , respectively, financed by a mix of taxes ($\gamma = 0.5$), for different degrees of fiscal policy coordination, comparing the

Figure 11: Fiscal Policy Coordination - Fiscal Shock in Country H

three scenarios of Pure Currency Union, Coordinated Currency Union, and Full Fiscal Union.

Looking at Figure 11, we see that the transmission of a fiscal shock acts through the consequent tax and price adjustments. In the Pure Currency Union scenario a decrease in public spending in country H allows to reduce taxes, which spurs private consumption and consequently domestic output. Higher private consumption has also inflationary effects, which brings net exports down. The drop in net exports then concurs to revert the path of real output that goes below its steady state value, and then back above it after net exports become positive. The dynamics of foreign variables follow closely the domestic ones due to the presence of complete financial markets.

In the other two scenarios (Coordinated Currency Union and Full Fiscal Union), where government consumption targets net exports, the international transmission of the fiscal policy shock is much more muted, since the terms of trade and consequently net exports are much more stable by construction. As a consequence, the overall volatility is consistently reduced with respect to the Pure Currency Union scenario. The dynamics in the Full Fiscal Union scenario are more stabilized compared to the Coordinated Currency Union scenario because of the international stabilization brought about by consolidating budget constraints

Figure 12: Fiscal Policy Coordination - Fiscal Shock in Country F

and distributing tax distortions. This implies that fiscal shocks are best managed by both targeting net exports and consolidating budget constraints, because they jointly push most variables to be more stable over time, counteracting the spillovers coming from fiscal policy shocks. At the same time, although it is not so clear which of the two dimensions is more important for stabilization purposes, fiscal policy can take advantage of both the benefits from coordinating on the same target (net exports) and those from consolidating budget constraints, so that if for some reason the stabilizing power of one of the two is missing, the other can make up for it.

The same conclusions about the stabilizing properties of a regime that targets net exports and consolidates budget constraints can be drawn by looking at Figure 12. Notice that, however, a negative fiscal policy shock in country F reduces domestic and foreign output, consequently increasing overall taxes. These opposite dynamics are mainly due to the structural differences between the two countries and, in particular, due to the higher price stickiness and lower trade openness of country F , that decreases the sensitivity of output and consumption to international adjustments. As a consequence, net exports increase substantially to compensate for the weaker response of producer prices in country F . The surge in net exports, however, is

not sufficient to increase output in country H , because of the opposite effect that taxation has on private consumption and output. These findings are consistent with the different signs of the fiscal policy multipliers in country H and in country F .

7 The Case for International Goods as Complements

In the discussion above we highlighted the role played by international demand gaps in shaping the transmission of either demand, supply or fiscal shocks, where the substitution effect of a price change dominates the income effect. However, [Hjortsø \(2012\)](#) shows that this latter result holds only in the case of a high international trade elasticity (η), while the opposite holds in the case of a low international trade elasticity, which is when goods are complements ($\eta \leq \frac{1}{\sigma}$). More importantly, this affects the ability of fiscal policy to reduce international imbalances, like fluctuations in the terms of trade and in net exports. For these reasons, we assess how much the previous results are sensitive to the alternative assumption of international goods as complements, instead of substitutes.

7.1 Fiscal Policy Coordination and Welfare: a Comparison

Figure 13 shows the impulse responses to a negative technology shock in country H in the Coordinated Currency Union scenario and in the Full Fiscal Union one, comparing the two cases of international goods as substitutes and complements. When goods are substitutes PPI inflation becomes relatively higher in country H compared to country F . As a result the terms of trade deteriorate and the substitution effect dominates the income effect, lowering net exports and output. When domestic and foreign goods are instead complements, the income effect dominates the substitution effect, spurring net exports and output in country H .

Notice that in the Full Fiscal Union scenario the overall volatility of the economy is substantially lower, in particular that of output and net exports. This finding is consistent with [Hjortsø \(2012\)](#), who highlights the fact that the cross-country insurance role of fiscal policy is smaller when the traded goods are substitutes instead of complements. More in detail, Figure 13 shows that, although with substitute goods the dynamics are less amplified altogether, with complement goods the Full Fiscal Union scenario produces quite more stabilization than the Coordinated Currency Union scenario.

The domination of the income effect on the substitution effect could also affect the welfare analysis, both in terms of Consumption Equivalent Variations and in terms of an ad hoc loss function. When the traded goods are complements, relative prices react more aggressively to shocks in response to international demand imbalances, and the welfare gains in targeting net exports are quite inferior.

Figure 13: Substitutes vs Complements - Technology Shock in Country H

As Table 5 reports, the highest welfare gains in terms of Consumption Equivalent Variations are attained in the Coordinated Currency Union scenario, but since the income effect dominates the substitution effect, the largest welfare gains from demand stabilization are for country F . Even if there is a welfare gain from stabilizing both the output gap and the net exports gap with a low elasticity of substitution, the income effect and the substitution effect move in the same direction for the country featuring the highest price rigidity (country F), while they move in opposite directions for country H . Therefore, since the stabilization of net exports partially offsets the substitution effect, there are additional welfare gains for country F from stabilizing international demand imbalances.

Finally, Table 5 shows that the consolidation of budget constraints leads to welfare losses, also when the elasticity of substitution is low. Additionally, stabilizing net exports, when budget constraints are consolidated, amplifies the effects of distributed tax adjustments, because it offsets price adjustments that would otherwise counterbalance the tax adjustments, bringing to welfare losses for country F .

Table 5: Optimal Fiscal Policy Parameters and Welfare Gains based on CEV

Policy Scenarios	Optimal Parameters*		Conditional Welfare Gains		
	ψ	ψ^*	Country H	Country F	Total
PCU	0	0	0%	0%	0%
PCU	0.0692	0.0373	0.01%	0.04%	0.03%
CCU	0.0729	0.0373	0.02%	0.23%	0.15%
FFU	0.0729	0.0373	0.02%	-0.40%	-0.23%
FFU	0	0	0.02%	-0.12%	-0.06%

*The optimal parameters are selected looking at the maximum unconditional expected lifetime utility.

Table 6: Welfare Gains based on ad hoc loss function

Policy Scenario	Loss Functions			Welfare Gains*		
	Country H	Country F	Total	Country H	Country F	Total
PCU ($\psi = \psi^* = 0$)	0.0226	0.0220	0.0222	0%	0%	0%
PCU	0.0222	0.0215	0.0218	1.62%	2.06%	1.88%
CCU	0.2228	0.2029	0.2108	-887%	-822%	-849%
FFU	0.1132	0.1015	0.1062	-401%	-361%	-377%
FFU ($\psi = \psi^* = 0$)	0.0382	0.0375	0.0378	-69.31%	-170.54%	-70.04%

* Welfare Gains are computed as $\frac{Loss_b - Loss_a}{Loss_b}$, with $Loss_b$ the loss of the PCU with $\psi = \psi^* = 0$.

Since there is no trade off between price and output stabilization if the internationally traded goods are complements, fiscal policy is able to successfully stabilize the output gap. For these reasons, Table 6 reports that only in the Pure Currency Union scenario with countercyclical government consumption there is a gain in terms of an ad hoc loss function, compared to the case of exogenous government consumption. Since with a low international trade elasticity there is a trade off between output stabilization and international demand stabilization, targeting net exports has a cost in terms of both output and price stabilization for both countries, which is why there are welfare losses in most cases. Finally, consolidating budgets partially offsets the amplification effect due to the stabilization of net exports, which is why the Full Fiscal Union scenario, despite being less stabilizing than the Pure Currency Union scenario, has a lower cost in terms of the ad hoc loss function than the Coordinated Currency Union scenario, as can be observed in Table 6.

7.2 Fiscal Shocks and Fiscal Multipliers and Spillovers

Hjortso (2012) argues that when goods are complements fiscal policy is more effective on consumption and output. In Figure 14 we document similar evidence for the case of a fiscal tightening in country H . Private consumption and output are more volatile, with the effect of the initial drop in the PPI amplified by the reduction in net exports for country H , due to the substitution effect. On the other hand, a negative fiscal shock in country F , shown in Figure 15, triggers a higher output in country H due to the increase in net exports. Consistently with the different reactions of net exports after a negative fiscal policy shock in

Figure 14: Substitutes vs Complements - Fiscal Shock in Country H

countries H and F , Figures 16 and 17 show that the sign of the fiscal policy multipliers do not only depend on the degree of fiscal policy coordination, but also on the international trade elasticity. The fiscal policy multiplier is positive for country H with a high international trade elasticity. Moreover, also price stickiness plays a relevant role since it determines the sign of the substitution effect, while the international trade elasticity determines which dominates between the substitution effect and the income effect.

In Figures 14 and 15 we observe the opposite reactions of net exports after a fiscal policy shock, while from Figures 16 and 17 it is clear that the effect of a fiscal shock on output is the opposite if the internationally traded goods are complements rather than substitutes. Since this fact implies a positive fiscal policy multiplier for country H when international goods are complements and a negative one when international goods are substitutes, while for country F it is the contrary, we can infer that the fiscal policy multiplier depends strongly and non-linearly on the relative degree of openness to international trade and on the international trade elasticity. Indeed, together they determine the sign of the substitution effect and whether the substitution effect or the income effect dominates.

Figure 15: Substitutes vs Complements - Fiscal Shock in Country F

8 Conclusions and Possible Extensions

This paper provides a characterization of the stabilization properties of different fiscal policy scenarios in a Two-Country Open-Economy New-Keynesian DSGE model of a Currency Union. We consider three fiscal policy scenarios. In the Pure Currency Union scenario, each fiscal authority moves government spending counter-cyclically in response to deviations of real output from steady state. In the other two scenarios – Coordinated Currency Union and Full Fiscal Union respectively – each government targets pro-cyclically a common variable, the net exports gap, but in the Full Fiscal Union scenario the two government budget constraints are consolidated and the tax rates in the two countries move together to finance the government expenditure. In all three scenarios, the fiscal authorities have access only to distortionary taxation and must balance the budget. The presence of distortionary taxation and price rigidity implies that fiscal policy and monetary policy are interconnected and produce real effects in the economy.

The main difference between our work and the previous literature is that we consider jointly the sta-

Figure 16: Fiscal Multiplier in Country H

bilization properties of targeting rules for fiscal policy and its financing with distortionary taxation, while most literature assumes stochastic government spending and lump-sum taxes, which implies very different dynamics. Based on the strength of the targeting rule, distortionary taxation might reverse the qualitative effects of fiscal policy on the economy, bringing to very different results.

Our main result is that coordinating fiscal policy by targeting the net exports gap produces much more stabilization in the economy than targeting the output gap, which provides even less stabilization on output itself. By also consolidating budget constraints we attain the most stabilized dynamics between countries through the joint adjustment of the tax rates. Actually, even only consolidating budget constraints with stochastic government spending produces about as much stabilization as only targeting the net exports gap without consolidating budget constraints, which implies that adding one dimension to the other barely produces more stabilization. This can be viewed as two instruments for stabilization that can be used jointly, while one can make up for the temporary lack of the other, as a sort of insurance mechanism.

Our analysis also highlights that taxes on wage income entail exponentially more distortionary effects

Figure 17: Fiscal Multiplier in Country F

than taxes on firm sales, which implies that the latter should be generally preferred to the former. This has a significant effect on the sign of fiscal policy multipliers and spillovers. A fiscal expansion generally implies a negative multiplier in the Core country of our model, which is characterized by lower price rigidity and trade openness. On the contrary, the multiplier is positive in the Periphery country, where prices are stickier and a fiscal expansion induces a deterioration in the terms of trade, fostering bigger consumption and output, which then spills over to the other country through complete financial markets. This also implies that the signs of the fiscal spillovers on the other country are the same as the signs of the fiscal multipliers in the first country.

In a robustness check, we also show that if the internationally traded goods are considered as complements, instead of substitutes, most of the dynamic effects of the shocks on the economy are reversed, consistently with the previous literature. Compared to the case of substitutes, the Full Fiscal Union scenario improves substantially the stability of the economy with respect to the Coordinated Currency Union, which is actually destabilizing in the case of complements, compared to the Pure Currency Union scenario. Consolidating

budget constraints allows to partially offset this destabilization produced by targeting the net exports gap. Although with international goods as complements the dynamics of the economy are reversed, our result that the Full Fiscal Union scenario is more stabilizing than the Coordinated Currency Union scenario still holds and to a greater extent compared to the case of substitutes.

Our model and results have nonetheless some shortcomings, which also entail possible future avenues of research. We assume complete international financial markets, which is a quite strong assumption, and more of an objective than a reality. Nonetheless, if fiscal policy has a cross-country insurance role with *complete* international financial markets, it must have an even stronger insurance role with *incomplete* international financial markets, where private cross-country insurance is no more available. We do not take into consideration explicitly the Zero Lower Bound on the nominal interest rate, which is an important feature of the recent global liquidity trap. In this case too, if fiscal policy affects the economy without constraints on monetary policy, with these constraints fiscal policy should be even more effective. Last but not least, we allow for positive government debt, but without allowing it to vary over time. Allowing for government debt dynamics would modify the evolution of most variables in the economy and would influence the government budget and the effects of fiscal policy. Also constraining households' access to financial markets would amplify the effects of fiscal policy, as households' consumption would be much more dependent on current income rather than permanent income.

This research was conducted to assess the stabilization properties of fiscal policy coordination in the Eurozone and to bring to policy conclusions for the proper macroeconomic management of a Currency Union. The policy prescriptions are that countries in the Eurozone should reduce international demand imbalances by either stabilizing trade flows across countries or by creating some form of fiscal union with a common budget and taxation strategy, or both, while avoiding the excessive use of labour taxes, in favour of sales taxes. This would dampen all shocks hitting different countries of the Eurozone and would reduce the volatility of output and consumption in every country. A future Fiscal Union would also need a Eurozone treasury for collective decision-making on fiscal policy, which would need to be accountable and legitimated democratically, but this is out of the scope of the present paper.

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A Mathematical Appendix

Here we provide all the equilibrium conditions and steady state values of the model.

A.1 Equilibrium Conditions

Here we collect all the equilibrium conditions of the full model, differentiating between a pure Currency Union, a Coordinated Currency Union and a Full Fiscal Union and between different policy rules.

The equilibrium conditions of the model are grouped into the following blocks:

Aggregate Demand Block

The aggregate demand block is composed of aggregate demand in both countries H :

$$Y_t = [1 - \alpha + \alpha(\mathcal{S}_t)^{1-\eta}]^{\frac{\eta}{1-\eta}} \left[1 - \alpha + \alpha^* \frac{1-h}{h} \left(\frac{\xi_t^*}{\xi_t} \right)^{\frac{1}{\sigma}} (\mathcal{S}_t)^{\eta-\frac{1}{\sigma}} \left(\frac{1 - \alpha^* + \alpha^*(\mathcal{S}_t)^{\eta-1}}{1 - \alpha + \alpha(\mathcal{S}_t)^{1-\eta}} \right)^{\frac{1-\eta\sigma}{\sigma(1-\eta)}} \right] C_t + G_t \quad (\text{A.1.1})$$

and F :

$$Y_t^* = [1 - \alpha^* + \alpha^*(\mathcal{S}_t)^{\eta-1}]^{\frac{\eta}{1-\eta}} \left[1 - \alpha^* + \alpha^* \frac{h}{1-h} \left(\frac{\xi_t}{\xi_t^*} \right)^{\frac{1}{\sigma}} (\mathcal{S}_t)^{\frac{1}{\sigma}-\eta} \left(\frac{1 - \alpha^* + \alpha^*(\mathcal{S}_t)^{\eta-1}}{1 - \alpha + \alpha(\mathcal{S}_t)^{1-\eta}} \right)^{\frac{1-\eta\sigma}{\sigma(1-\eta)}} \right] C_t^* + G_t^* \quad (\text{A.1.2})$$

while the evolution of private consumption is given by the households' Euler Equation in country F :

$$\frac{1}{1+i_t} = \beta E_t \left\{ \frac{\xi_{t+1}^*}{\xi_t^*} \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \frac{1}{\Pi_{t+1}^*} \right\} \quad (\text{A.1.3})$$

and by the international risk-sharing condition in country H :

$$C_t = \frac{h}{1-h} \left[\frac{\xi_t}{\xi_t^*} \mathcal{S}_t \left(\frac{1 - \alpha^* + \alpha^*(\mathcal{S}_t)^{\eta-1}}{1 - \alpha + \alpha(\mathcal{S}_t)^{1-\eta}} \right)^{\frac{1}{1-\eta}} \right]^{\frac{1}{\sigma}} C_t^* \quad (\text{A.1.4})$$

while the relationship between CPI inflation and PPI inflation is given by:

$$\Pi_t = \Pi_{H,t} \left[\frac{1 - \alpha + \alpha(\mathcal{S}_t)^{1-\eta}}{1 - \alpha + \alpha(\mathcal{S}_{t-1})^{1-\eta}} \right]^{\frac{1}{1-\eta}} \quad (\text{A.1.5})$$

in country H and:

$$\Pi_t^* = \Pi_{H,t}^* \left[\frac{1 - \alpha^* + \alpha^*(\mathcal{S}_t)^{\eta-1}}{1 - \alpha^* + \alpha^*(\mathcal{S}_{t-1})^{\eta-1}} \right]^{\frac{1}{1-\eta}} \quad (\text{A.1.6})$$

in country F , and the evolution of the terms of trade is given by:

$$\mathcal{S}_t = \frac{\Pi_{H,t}^*}{\Pi_{H,t}} \mathcal{S}_{t-1} \quad (\text{A.1.7})$$

while the exogenous demand shocks evolve according to:

$$\xi_t = (\xi_{t-1})^{\rho_\xi} e^{\varepsilon_t} \quad (\text{A.1.8})$$

$$\xi_t^* = (\xi_{t-1}^*)^{\rho_\xi^*} e^{\varepsilon_t^*} \quad (\text{A.1.9})$$

Aggregate Supply Block

The aggregate supply block is composed of the aggregate supply equation for country H :

$$\left(\frac{1 - \theta(\Pi_{H,t})^{\varepsilon-1}}{1 - \theta} \right)^{\frac{1}{1-\varepsilon}} = \frac{\varepsilon}{\varepsilon - 1} \frac{K_t}{F_t} \quad (\text{A.1.10})$$

where:

$$K_t = \xi_t (C_t)^{-\sigma} Y_t MC_t + \beta \theta E_t \left\{ \frac{(\Pi_{H,t+1})^{\varepsilon+1}}{\Pi_{t+1}} K_{t+1} \right\} \quad (\text{A.1.11})$$

$$F_t = \xi_t (C_t)^{-\sigma} Y_t (1 - \tau_t^s) + \beta \theta E_t \left\{ \frac{(\Pi_{H,t+1})^\varepsilon}{\Pi_{t+1}} F_{t+1} \right\} \quad (\text{A.1.12})$$

and marginal cost in country H is given by:

$$MC_t = \frac{(Y_t)^\varphi (d_t)^\varphi (C_t)^\sigma}{(1 - \tau_t^w)(A_t)^{1+\varphi} (h)^\varphi} [1 - \alpha + \alpha(\mathcal{S}_t)^{1-\eta}]^{\frac{1}{1-\eta}} \quad (\text{A.1.13})$$

and the aggregate supply equation for country F :

$$\left(\frac{1 - \theta^*(\Pi_{H,t}^*)^{\varepsilon-1}}{1 - \theta^*} \right)^{\frac{1}{1-\varepsilon}} = \frac{\varepsilon}{\varepsilon - 1} \frac{K_t^*}{F_t^*} \quad (\text{A.1.14})$$

where:

$$K_t^* = \xi_t^* (C_t^*)^{-\sigma} Y_t^* MC_t^* + \beta \theta^* E_t \left\{ \frac{(\Pi_{H,t+1}^*)^{\varepsilon+1}}{\Pi_{t+1}^*} K_{t+1}^* \right\} \quad (\text{A.1.15})$$

$$F_t^* = \xi_t^* (C_t^*)^{-\sigma} Y_t^* (1 - \tau_t^{*s}) + \beta \theta^* E_t \left\{ \frac{(\Pi_{H,t+1}^*)^\varepsilon}{\Pi_{t+1}^*} F_{t+1}^* \right\} \quad (\text{A.1.16})$$

and marginal cost in country F is given by:

$$MC_t^* = \frac{(Y_t^*)^\varphi (d_t^*)^\varphi (C_t^*)^\sigma}{(1 - \tau_t^{*w})(A_t^*)^{1+\varphi}(1-h)^{\varphi+\sigma}} [1 - \alpha^* + \alpha^*(\mathcal{S}_t)^{\eta-1}]^{\frac{1}{1-\eta}} \quad (\text{A.1.17})$$

while the evolution of price dispersion is given by:

$$d_t = \theta d_{t-1}(\Pi_{H,t})^\varepsilon + (1 - \theta) \left[\frac{1 - \theta(\Pi_{H,t})^{\varepsilon-1}}{1 - \theta} \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad (\text{A.1.18})$$

for country H , and:

$$d_t^* = \theta^* d_{t-1}^*(\Pi_{H,t}^*)^\varepsilon + (1 - \theta^*) \left[\frac{1 - \theta^*(\Pi_{H,t}^*)^{\varepsilon-1}}{1 - \theta^*} \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad (\text{A.1.19})$$

for country F , while the levels of technology evolve exogenously according to:

$$A_t = (A_{t-1})^{\rho_a} e^{\varepsilon_t} \quad (\text{A.1.20})$$

$$A_t^* = (A_{t-1}^*)^{\rho_a^*} e^{\varepsilon_t} \quad (\text{A.1.21})$$

Net Exports, Net Foreign Assets and the Balance of Payments

Real Net Exports for country H are given by:

$$\widetilde{NX}_t = Y_t - [1 - \alpha + \alpha(\mathcal{S}_t)^{1-\eta}]^{\frac{1}{1-\eta}} C_t - G_t \quad (\text{A.1.22})$$

Real Net Foreign Assets for country H are given by:

$$\widetilde{NFA}_t = \tilde{D}_t + \tilde{B}_t - \tilde{B}_t^G \quad (\text{A.1.23})$$

The real Balance of Payments for country H is given by:

$$\widetilde{BP}_t \equiv \widetilde{NX}_t + \frac{i_{t-1}}{\Pi_{H,t}} \widetilde{NFA}_{t-1} \quad (\text{A.1.24})$$

so that real Net Foreign Assets for country H evolve according to:

$$\widetilde{NFA}_t = \frac{\widetilde{NFA}_{t-1}}{\Pi_{H,t}} + \widetilde{BP}_t \quad (\text{A.1.25})$$

Monetary Policy

Monetary policy sets the nominal interest rate following the rule:

$$\beta(1 + i_t) = \left(\frac{\Pi_t^U}{\Pi^U} \right)^{\phi_\pi(1-\rho_i)} [\beta(1 + i_{t-1})]^{\rho_i} \quad (\text{A.1.26})$$

where union-wide CPI inflation is defined by:

$$\Pi_t^U \equiv (\Pi_t)^h (\Pi_t^*)^{1-h} \quad (\text{A.1.27})$$

Fiscal Policy in a Pure Currency Union - Consumption Scenario

Fiscal policy, in the consumption scenario of the Pure Currency Union setting, sets government consumption following the rule:

$$\frac{G_t}{G} = \left(\frac{Y_t}{Y} \right)^{-\psi_y(1-\rho_g)} \left(\frac{G_{t-1}}{G} \right)^{\rho_g} e^{\varepsilon_t} \quad (\text{A.1.28})$$

for country H , and:

$$\frac{G_t^*}{G^*} = \left(\frac{Y_t^*}{Y^*} \right)^{-\psi_y^*(1-\rho_g^*)} \left(\frac{G_{t-1}^*}{G^*} \right)^{\rho_g^*} e^{\varepsilon_t} \quad (\text{A.1.29})$$

for country F , while keeping real transfers constant and balancing the budget:

$$\tilde{T}_t = \tilde{T} \quad \tilde{B}_t^G = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \quad \text{and} \quad \tilde{T}_t^* = \tilde{T}^* \quad \tilde{B}_t^{*G} = \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} \quad (\text{A.1.30})$$

so financing fiscal policy by the variation of the tax rates on wage income and firm sales from their steady state levels respectively by a share $\gamma \in [0, 1]$ ($\gamma^* \in [0, 1]$ for country F) and $1 - \gamma$ ($1 - \gamma^*$ for country F) through the following tax rules:

$$\gamma(\tau_t^s - \tau^s) = (1 - \gamma)(\tau_t^w - \tau^w) \quad (\text{A.1.31})$$

$$\gamma^*(\tau_t^{*s} - \tau^{*s}) = (1 - \gamma^*)(\tau_t^{*w} - \tau^{*w}) \quad (\text{A.1.32})$$

with the following budget constraints:

$$G_t + \tilde{T}_t + i_{t-1} \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} = \tau_t^s Y_t + \tau_t^w MC_t d_t Y_t + \tilde{B}_t^G - \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \quad (\text{A.1.33})$$

$$G_t^* + \tilde{T}_t^* + i_{t-1} \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} = \tau_t^{*s} Y_t^* + \tau_t^{*w} MC_t^* d_t^* Y_t^* + \tilde{B}_t^{*G} - \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} \quad (\text{A.1.34})$$

Fiscal Policy in a Pure Currency Union - Transfer Scenario

In the transfer scenario of the Pure Currency Union setting, fiscal policy sets real transfers following the rule:

$$\frac{\tilde{T}_t}{\tilde{T}} = \left(\frac{Y_t}{Y} \right)^{-\psi_y(1-\rho_t)} \left(\frac{\tilde{T}_{t-1}}{\tilde{T}} \right)^{\rho_t} e^{\varepsilon_t} \quad (\text{A.1.35})$$

for country H , and:

$$\frac{\tilde{T}_t^*}{\tilde{T}^*} = \left(\frac{Y_t^*}{Y^*} \right)^{-\psi_y^*(1-\rho_t^*)} \left(\frac{\tilde{T}_{t-1}^*}{\tilde{T}^*} \right)^{\rho_t^*} e^{\varepsilon_t} \quad (\text{A.1.36})$$

for country F , while keeping government consumption constant and balancing the budget:

$$G_t = G \quad \tilde{B}_t^G = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \quad \text{and} \quad G_t^* = G^* \quad \tilde{B}_t^{*G} = \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} \quad (\text{A.1.37})$$

Fiscal Policy in a Coordinated Currency Union - Consumption Scenario

Fiscal policy, in the consumption scenario of the Coordinated Currency Union setting, sets government consumption following the rule:

$$\frac{G_t}{G} = \left(\frac{\widetilde{NX}_t}{\widetilde{NX}} \right)^{\psi_{nx}(1-\rho_g)} \left(\frac{G_{t-1}}{G} \right)^{\rho_g} e^{\varepsilon_t} \quad (\text{A.1.38})$$

for country H , and:

$$\frac{G_t^*}{G^*} = \left(\frac{S \widetilde{NX}_t}{S_t \widetilde{NX}} \right)^{-\psi_{nx}^*(1-\rho_g^*)} \left(\frac{G_{t-1}^*}{G^*} \right)^{\rho_g^*} e^{\varepsilon_t} \quad (\text{A.1.39})$$

for country F , while keeping real transfers constant and balancing the budget:

$$\tilde{T}_t = \tilde{T} \quad \tilde{B}_t^G = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \quad \text{and} \quad \tilde{T}_t^* = \tilde{T}^* \quad \tilde{B}_t^{*G} = \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} \quad (\text{A.1.40})$$

so financing fiscal policy by the variation of the tax rates on wage income and firm sales from their steady state levels respectively by a share $\gamma \in [0, 1]$ ($\gamma^* \in [0, 1]$ for country F) and $1 - \gamma$ ($1 - \gamma^*$ for country F) through the following tax rules:

$$\gamma(\tau_t^s - \tau^s) = (1 - \gamma)(\tau_t^w - \tau^w) \quad (\text{A.1.41})$$

$$\gamma^*(\tau_t^{*s} - \tau^{*s}) = (1 - \gamma^*)(\tau_t^{*w} - \tau^{*w}) \quad (\text{A.1.42})$$

with the following budget constraints:

$$G_t + \tilde{T}_t + i_{t-1} \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} = \tau_t^s Y_t + \tau_t^w MC_t d_t Y_t + \tilde{B}_t^G - \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \quad (\text{A.1.43})$$

$$G_t^* + \tilde{T}_t^* + i_{t-1} \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} = \tau_t^{*s} Y_t^* + \tau_t^{*w} M C_t^* d_t^* Y_t^* + \tilde{B}_t^{*G} - \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} \quad (\text{A.1.44})$$

Fiscal Policy in a Coordinated Currency Union - Transfer Scenario

In the transfer scenario of the Coordinated Currency Union setting, fiscal policy sets real transfers following the rule:

$$\frac{\tilde{T}_t}{\tilde{T}} = \left(\frac{\widetilde{NX}_t}{\widetilde{NX}} \right)^{\psi_{nx}(1-\rho_t)} \left(\frac{\tilde{T}_{t-1}}{\tilde{T}} \right)^{\rho_t} e^{\varepsilon_t} \quad (\text{A.1.45})$$

for country H , and:

$$\frac{\tilde{T}_t^*}{\tilde{T}^*} = \left(\frac{S \widetilde{NX}_t}{S_t \widetilde{NX}} \right)^{-\psi_{nx}^*(1-\rho_t^*)} \left(\frac{\tilde{T}_{t-1}^*}{\tilde{T}^*} \right)^{\rho_t^*} e^{\varepsilon_t} \quad (\text{A.1.46})$$

for country F , while keeping government consumption constant and balancing the budget:

$$G_t = G \quad \tilde{B}_t^G = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \quad \text{and} \quad G_t^* = G^* \quad \tilde{B}_t^{*G} = \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} \quad (\text{A.1.47})$$

Fiscal Policy in a Full Fiscal Union - Consumption Scenario

Fiscal policy, in the consumption scenario of the Full Fiscal Union setting, sets government consumption following the rules:

$$\frac{G_t}{G} = \left(\frac{\widetilde{NX}_t}{\widetilde{NX}} \right)^{\psi_{nx}(1-\rho_g)} \left(\frac{G_{t-1}}{G} \right)^{\rho_g} e^{\varepsilon_t} \quad (\text{A.1.48})$$

$$\frac{G_t^*}{G^*} = \left(\frac{S \widetilde{NX}_t}{S_t \widetilde{NX}} \right)^{-\psi_{nx}^*(1-\rho_g^*)} \left(\frac{G_{t-1}^*}{G^*} \right)^{\rho_g^*} e^{\varepsilon_t} \quad (\text{A.1.49})$$

while keeping real transfers constant and balancing the overall budget:

$$\tilde{T}_t = \tilde{T} \quad \tilde{T}_t^* = \tilde{T}^* \quad \text{and} \quad \tilde{B}_t^G = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} \implies \tilde{B}_t^G + S_t \tilde{B}_t^{*G} = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} + S_t \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} \quad (\text{A.1.50})$$

so financing fiscal policy by the variation of the tax rates on wage income and firm sales from their steady state levels respectively by a share $\gamma \in [0, 1]$ and $1 - \gamma$, while distributing equally among the two countries the cost of fiscal policy, through the following tax rules:

$$\gamma(\tau_t^s - \tau^s) = (1 - \gamma)(\tau_t^w - \tau^w) \quad (\text{A.1.51})$$

$$(\tau_t^s - \tau^s) = (\tau_t^{*s} - \tau^{*s}) \quad (\text{A.1.52})$$

$$(\tau_t^w - \tau^w) = (\tau_t^{*w} - \tau^{*w}) \quad (\text{A.1.53})$$

with the following consolidated budget constraint:

$$G_t + \tilde{T}_t + S_t(G_t^* + \tilde{T}_t^*) + i_{t-1} \frac{\tilde{\mathcal{B}}_{t-1}^G}{\Pi_{H,t}} = (\tau_t^s + \tau_t^w MC_t d_t) Y_t + (\tau_t^{*s} + \tau_t^{*w} MC_t^* d_t^*) S_t Y_t^* + \tilde{\mathcal{B}}_t^G - \frac{\tilde{\mathcal{B}}_{t-1}^G}{\Pi_{H,t}} \quad (\text{A.1.54})$$

Fiscal Policy in a Full Fiscal Union - Transfer Scenario

In the transfer scenario of the Full Fiscal Union setting, fiscal policy sets real transfers following the rules:

$$\frac{\tilde{T}_t}{\tilde{T}} = \left(\frac{\widetilde{NX}_t}{\widetilde{NX}} \right)^{\psi_{nx}(1-\rho_t)} \left(\frac{\tilde{T}_{t-1}}{\tilde{T}} \right)^{\rho_t} e^{\varepsilon_t} \quad (\text{A.1.55})$$

$$\frac{\tilde{T}_t^*}{\tilde{T}^*} = \left(\frac{S \widetilde{NX}_t}{S_t \widetilde{NX}} \right)^{-\psi_{nx}^*(1-\rho_t^*)} \left(\frac{\tilde{T}_{t-1}^*}{\tilde{T}^*} \right)^{\rho_t^*} e^{\varepsilon_t} \quad (\text{A.1.56})$$

while keeping government consumption constant and balancing the overall budget:

$$G_t = G \quad G_t^* = G^* \quad \text{and} \quad \tilde{\mathcal{B}}_t^G = \frac{\tilde{\mathcal{B}}_{t-1}^G}{\Pi_{H,t}} \implies \tilde{B}_t^G + S_t \tilde{B}_t^{*G} = \frac{\tilde{B}_{t-1}^G}{\Pi_{H,t}} + S_t \frac{\tilde{B}_{t-1}^{*G}}{\Pi_{H,t}^*} \quad (\text{A.1.57})$$

We can now define an equilibrium for the Currency Union.

Definition 1 (Equilibrium). *An Imperfectly competitive equilibrium is a sequence of stochastic processes $\mathcal{X}_t \equiv \{Y_t, Y_t^*, C_t, C_t^*, \Pi_{H,t}, \Pi_{H,t}^*, \Pi_t, \Pi_t^*, \Pi_t^U, S_t, K_t, K_t^*, F_t, F_t^*, MC_t, MC_t^*, d_t, d_t^*, \widetilde{NX}_t, \widetilde{NFA}_t, \widetilde{CA}_t\}$ and exogenous disturbances $\mathcal{Z}_t \equiv \{\xi_t, \xi_t^*, A_t, A_t^*\}$ satisfying equations [A.1.1](#) through [A.1.25](#) and the definition of union-wide inflation [A.1.27](#), given initial conditions $\mathcal{I}_{-1} \equiv \{C_{-1}, C_{-1}^*, \Pi_{H,-1}, \Pi_{H,-1}^*, S_{-1}, d_{-1}, d_{-1}^*, \widetilde{NFA}_{-1}\}$, plus monetary and fiscal policies $\mathcal{P}_t \equiv \{i_t, G_t, G_t^*, \tilde{T}_t, \tilde{T}_t^*, \tau_t^s, \tau_t^{*s}, \tau_t^w, \tau_t^{*w}, \tilde{B}_t^G, \tilde{B}_t^{*G}\}$ specified in equation [A.1.26](#) for monetary policy and in equations [A.1.28](#) through [A.1.57](#) for the various specifications of fiscal policy, for $t \geq 0$.*

A.2 The Steady State

We describe the symmetric (in terms of per capita consumption and prices) non-stochastic steady state with constant government debt and zero inflation, which will be the starting point of our simulations. We focus on the perfect foresight steady state and equilibrium deviations from it, given by different shocks. *Perfect Foresight* is a viable assumption because, despite the uncertainty to which price-setters are subject, it disappears in the aggregate due to the further assumption that there is a large number (more accurately, a continuum) of firms, as explained in [Calvo \(1983\)](#).

The symmetric non-stochastic steady state with constant government debt and zero inflation, which will be

the starting point of our simulations, is defined by the following assumptions and equations.

All shocks are constant at their long-run levels of 1:

$$\xi = \xi^* = A = A^* = 1 \quad (\text{A.2.1})$$

There is no inflation and no price dispersion:

$$\Pi_H = \Pi_H^* = \Pi = \Pi^* = \Pi^U = 1 \implies d = d^* = 1 \quad (\text{A.2.2})$$

The terms of trade and the real exchange rate are equal to 1:

$$S = 1 \implies Q = 1 \quad (\text{A.2.3})$$

Per-capita consumption is equal across countries:

$$\frac{C}{h} = \frac{C^*}{1-h} \quad (\text{A.2.4})$$

Aggregate demand in each country is given by:

$$Y = \left(1 - \alpha + \alpha^* \frac{1-h}{h}\right) C + G \quad (\text{A.2.5})$$

$$Y^* = \left(1 - \alpha^* + \alpha \frac{h}{1-h}\right) C^* + G^* \quad (\text{A.2.6})$$

Combining the previous equations we can derive per-capita consumption in each country as a function of output and government spending and equate the two to derive an equation linking output and government spending in the two countries:

$$Y = \frac{(1-\alpha)h + \alpha^*(1-h)}{(1-\alpha^*)(1-h) + \alpha h} [Y^* - G^*] + G \quad (\text{A.2.7})$$

From the Euler Equations:

$$\frac{1}{1+i} = \beta \implies i = \frac{1}{\beta} - 1 \quad (\text{A.2.8})$$

Recalling marginal costs in steady state from price-setting:

$$MC = \frac{\varepsilon - 1}{\varepsilon} (1 - \tau^s) \quad (\text{A.2.9})$$

$$MC^* = \frac{\varepsilon - 1}{\varepsilon} (1 - \tau^{*s}) \quad (\text{A.2.10})$$

Marginal costs are also given by labour market equilibrium:

$$MC = \frac{(Y)^\varphi (C)^\sigma}{(1 - \tau^w)(h)^{\varphi + \sigma}} \quad (\text{A.2.11})$$

$$MC^* = \frac{(Y^*)^\varphi (C^*)^\sigma}{(1 - \tau^{*w})(1 - h)^{\varphi + \sigma}} \quad (\text{A.2.12})$$

Equating the two marginal cost expressions for each country yields consumption in terms of output:

$$C = \left[\frac{\varepsilon - 1}{\varepsilon} \frac{(1 - \tau^s)(1 - \tau^w)(h)^{\varphi + \sigma}}{(Y)^\varphi} \right]^{\frac{1}{\sigma}} \quad (\text{A.2.13})$$

$$C^* = \left[\frac{\varepsilon - 1}{\varepsilon} \frac{(1 - \tau^{*s})(1 - \tau^{*w})(1 - h)^{\varphi + \sigma}}{(Y^*)^\varphi} \right]^{\frac{1}{\sigma}} \quad (\text{A.2.14})$$

Deriving per-capita consumption in the two countries and equating the two yields an equation linking output in the two countries:

$$Y^* = \frac{1 - h}{h} \left[\frac{(1 - \tau^{*s})(1 - \tau^{*w})}{(1 - \tau^s)(1 - \tau^w)} \right]^{\frac{1}{\varphi}} Y \quad (\text{A.2.15})$$

Substituting foreign output from the supply equation A.2.15 into the demand equation A.2.7 yields a relationship linking domestic output to government consumption in both countries²⁷:

$$Y = \left[1 - \frac{(1 - \alpha)h + \alpha^*(1 - h)}{(1 - \alpha^*)(1 - h) + \alpha h} \frac{1 - h}{h} \left[\frac{(1 - \tau^{*s})(1 - \tau^{*w})}{(1 - \tau^s)(1 - \tau^w)} \right]^{\frac{1}{\varphi}} \right]^{-1} \left[G - \frac{(1 - \alpha)h + \alpha^*(1 - h)}{(1 - \alpha^*)(1 - h) + \alpha h} G^* \right] \quad (\text{A.2.16})$$

Fiscal policy multipliers and spillovers in steady state for country H can be found from the previous equation by computing the effect on domestic output of variations of G and G^* respectively:

$$\frac{\partial Y}{\partial G} = \left[1 - \frac{(1 - \alpha)h + \alpha^*(1 - h)}{(1 - \alpha^*)(1 - h) + \alpha h} \frac{1 - h}{h} \left[\frac{(1 - \tau^{*s})(1 - \tau^{*w})}{(1 - \tau^s)(1 - \tau^w)} \right]^{\frac{1}{\varphi}} \right]^{-1} \quad (\text{A.2.17})$$

$$\frac{\partial Y}{\partial G^*} = - \left[\frac{(1 - \alpha)h + \alpha^*(1 - h)}{(1 - \alpha^*)(1 - h) + \alpha h} \right] \left[1 - \frac{(1 - \alpha)h + \alpha^*(1 - h)}{(1 - \alpha^*)(1 - h) + \alpha h} \frac{1 - h}{h} \left[\frac{(1 - \tau^{*s})(1 - \tau^{*w})}{(1 - \tau^s)(1 - \tau^w)} \right]^{\frac{1}{\varphi}} \right]^{-1} \quad (\text{A.2.18})$$

which shows how fiscal multipliers and spillovers depend on the parameters of the model, other than the tax rates in both countries.

We can see from the previous equations that a necessary and sufficient condition for fiscal multipliers to be

²⁷A specular equation can be derived linking foreign output to government consumption in both countries. This would allow to derive foreign fiscal multipliers and spillovers. The results and analysis would be symmetric with respect to the counterpart variables of the other country.

positive and greater than 1 and at the same time for fiscal spillovers to be negative is:

$$\frac{(1-\alpha)h + \alpha^*(1-h)}{(1-\alpha^*)(1-h) + \alpha h} \frac{1-h}{h} \left[\frac{(1-\tau^{*s})(1-\tau^{*w})}{(1-\tau^s)(1-\tau^w)} \right]^{\frac{1}{\varphi}} < 1 \quad (\text{A.2.19})$$

From the previous condition, sufficient, but not necessary, conditions for fiscal multipliers to be positive and greater than 1 and at the same time for fiscal spillovers to be negative are:

$$\frac{\alpha^*}{\alpha} < \frac{h}{1-h} \quad (\text{A.2.20})$$

$$\tau^{*s} + \tau^{*w} - \tau^{*s}\tau^{*w} > \tau^s + \tau^w - \tau^s\tau^w \quad (\text{A.2.21})$$

while, by changing the sign of all the inequalities, we get alternative conditions for fiscal multipliers to be negative and at the same time for fiscal spillovers to be positive and greater than 1. This tells us that when fiscal multipliers are positive and greater than 1, fiscal spillovers are negative, and viceversa.

Under the condition given by Equation A.2.19, fiscal multipliers in country H are positive and increasing with the openness to trade of the foreign country α^* , with the relative size of the foreign country $\frac{1-h}{h}$ and with the steady-state values of the domestic tax rates on firm sales τ^s and wage income τ^w , while they are positive and decreasing with the inverse of the Frisch elasticity of labour supply φ , with the openness to trade of the domestic country α and with the steady-state values of the foreign tax rates on firm sales τ^{*s} and wage income τ^{*w} . Under the same conditions, fiscal spillovers in country H are negative and increasing in absolute terms with the openness to trade of the foreign country α^* , with the relative size of the foreign country $\frac{1-h}{h}$ and with the steady-state values of the domestic tax rates on firm sales τ^s and wage income τ^w , while they are negative and decreasing in absolute terms with the inverse of the Frisch elasticity of labour supply φ , with the openness to trade of the domestic country α and with the steady-state values of the foreign tax rates on firm sales τ^{*s} and wage income τ^{*w} . In steady state real net exports are given by:

$$\widetilde{NX} = Y - C - G \quad (\text{A.2.22})$$

while real net foreign assets are:

$$\widetilde{NFA} = \tilde{D} + \tilde{B} - \tilde{B}^G \quad (\text{A.2.23})$$

The real balance of payments is given by:

$$\widetilde{BP} = \widetilde{NX} + \left(\frac{1}{\beta} - 1 \right) \widetilde{NFA} \quad (\text{A.2.24})$$

while from the budget constraints of households and governments, or equivalently from the evolution of net foreign assets:

$$\widetilde{NFA} = \widetilde{NFA} + \widetilde{BP} \quad (\text{A.2.25})$$

which implies that in steady state the balance of payments must be zero and so net exports pin down net foreign assets:

$$\widetilde{BP} = 0 \implies \widetilde{NX} = -\left(\frac{1}{\beta} - 1\right) \widetilde{NFA} \quad (\text{A.2.26})$$

These relations yield an equation linking output, government consumption and household consumption in the two countries in steady state, which mainly comes from the fact that net exports are in zero international net supply in steady state:

$$Y + Y^* = C + C^* + G + G^* \quad (\text{A.2.27})$$

The household budget constraints in steady state for countries H and F are given by:

$$C = \left(\frac{1}{\beta} - 1\right) (\tilde{D} + \tilde{B}) + \tilde{T} + Y(1 - \tau^s - \tau^w MC) \quad (\text{A.2.28})$$

$$C^* = \left(\frac{1}{\beta} - 1\right) (\tilde{D}^* + \tilde{B}^*) + \tilde{T}^* + Y^*(1 - \tau^{*s} - \tau^{*w} MC^*) \quad (\text{A.2.29})$$

Instead the government budget constraints of the two countries in steady state read:

$$G + \tilde{T} + \left(\frac{1}{\beta} - 1\right) \tilde{B}^G = Y(\tau^s + \tau^w MC) \quad (\text{A.2.30})$$

$$G^* + \tilde{T}^* + \left(\frac{1}{\beta} - 1\right) \tilde{B}^{*G} = Y^*(\tau^{*s} + \tau^{*w} MC^*) \quad (\text{A.2.31})$$