ON PRODUCT DIFFERENTIATION: OLD IDEAS VS. NEW TOOLS

Maurizio Di Cola∗

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1. Introduction
In industrial organization analysis one of the most studied subject is product differentiation. This kind of activity has numerous effects on the market. Through the production of several varieties of the same good firms can increase their profits and consumers can better satisfy their needs. However, in economic literature, it is also argued that product differentiation represents an impediment to entry in the market.

The aim of this work is to understand what product differentiation is. Indeed, through a deep analysis into economic literature on product differentiation we reconstruct the real role played by this strategy. First of all, we considered the demand side. Particularly, we tried to understand why consumers demand differentiated products. Secondly, we thought about the firm side, trying to underline which are the rationales for the firm to differentiate their products. From the supply side, this work explains how old theoretical models consider horizontal product differentiation an impediment to entry.

Using these models as starting point we tried to remove some basic assumptions with the aim to come close to real markets. Particularly, the first statement regard the difference between geographical and artificial spaces. Classical theoretical constructs on horizontal product differentiation treated geographical places as artificial ones. From this viewpoint this work considers consumers preferences as an artificial space. The awareness of the difference between geographical and artificial spaces leads to increase the dimensions of the characteristics space from only two to $n$-dimensions.

The description of an $n$-dimensions characteristics space allowed to introduce the powerful max-min principle, which states that firms use maximum differentiation along the $n$th characteristic and minimum differentiation along the $n-1$ characteristics.

The max-min principle open the door to a long waited conciliation in economic theory on product differentiation. Indeed, on one hand some old models found that firms are led by the agglomeration effect; they locate their

∗ Luiss Guido Carli University, mdicola@luiss.it

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products close to the ones produced by other firms with the plan to gain bigger market share. In other words, firms choose minimum differentiation. On the other hand, some scholars shown how firms are dominated by the strategic effect. They decide to locate their products as far as possible from the ones sold by their competitors, with the intend to exploit market power. Through the max-min principle the agglomeration and the strategic effect can live together. Firms choose the strategic effect in the $n^{th}$ characteristic and the agglomeration effect in the $n-1$ characteristics.

Finally, through games theory tools, specially with folk theorems we described how newcomers can enter the market in horizontally differentiated markets. We interpreted the entry process as an infinitely repeated game, where at each stage there could be a new entrant who wants to sell their products. This intuition is coherent with the market structure, where incumbent gains extra profits due to product differentiation. Therefore, newcomers are attracted by these additional earnings, and desire to enter the market. From this viewpoint the incumbent is playing an infinite entry game, every time with a different competitor.

2. The real meaning of product differentiation

Several scholars developed models to explain the functioning of markets with homogeneous goods, but in real markets homogeneity is the exception whilst differentiation is the rule. In each market it is possible to see firms selling differentiated products.

The aim to differentiate is to create market power; through product variety, firms can charge prices above marginal cost. But this is not the whole story, because through differentiation, social welfare could be enhanced. Consumers can have more varieties of the same good, then they can increase their surplus because they can find a good which is closer to their preferences; with an increase in the utility as final result. The gains from product diversity can be large and may easily outweigh the inefficiency costs resulting from downward sloping demand curves (Indic, 1998). Furthermore, product differentiation has a substantive effect of the firm growth. Through the production of several varieties, firms can sell their product to consumers who before did not buy the good, because they can meet consumers’ tastes in a better way.

The first useful distinction of the several types of product differentiation is between horizontal and vertical differentiation. The former case is when goods are differentiated only in their characteristics and not in quality. Goods have different tastes, colors and shapes, consumers can choose the closer good to their preferences. Vertical product differentiation concerns also the quality of goods. Firms decide to produce several goods with different qualities.
2.1. *Horizontal product differentiation in economic theory: the historical path*

A good starting point to study the economic theory on product differentiation could be the historical perspective.

The first distinction could be drawn between non-address and address branch (Eaton and Lipsey, 1989).

The birth of non-address branch could be identified with the Marshall’s model of perfect competition. Therefore, the first critics came from Sraffa (Sraffa, 1926) about the inconsistency between real world evidence where there are unexploited economies of scale in many manufacturing industries and the Marshallian theory of perfect competition, where scale economies are fully exploited in the long run.

The answer to that critics arrived from Chamberlin (Chamberlin, 1933). In his theory, there are a large group of firms producing differentiated products. He demonstrated how with free entry the equilibrium is when each firm’s output was less than minimum efficient scale. The relevance of this model was the introduction of the differentiation element, through that hypothesis was possible to explain the empirical result of unexploited economies of scale. The Chamberlin’s model was attacked in several aspects, one of these was the assumption of symmetric preferences. This postulation leads to a generalized competition where in one industry no firm had a near competitors. Under symmetric preferences, free entry drives profits to zero. Kaldor introduced the hypothesis of substitution between near products (Kaldor, 1934). In that view began the analysis of address model. He saw competition as localized, even within an industry.

By the 1960s, the interest on Chamberlin’s model decreased for two main reasons, first of all empirical results shown how differentiated industry are composed by few firms, where each of them produce several varieties. Secondly, location theory showed that localized, rather than generalized competition was also common in many industries where firms are differentiated in the geographic element.

Another interpretation of Chamberlin’s model came in the 1970s from Dixit and Stiglitz (Dixit and Stiglitz, 1977). They developed a model of monopolistic competition named representative consumer; where differences among consumers are ignored and all demands are proportional to those of a typical or standard consumer, who maximizes utility function\(^1\). The representative consumer is faced with a budget constraint and it is assumed to have a constant marginal utility of income (Ireland, 1987). This model was further developed by Perloff and Salop (Perloff and Salop, 1985), they considered several consumers with different tastes in a way to display the symmetric property.

\(^1\) The utility function is defined over amounts consumed of all commodities plus savings.
To build an historical path for address branch’s models is useful to consider the duopoly models developed by Cournot and Bertrand. The Cournot’s model explained the competition among firm in oligopoly on quantities (Bacon, 1963). Instead Bertrand considered the competition in oligopoly on price (Bertrand, 1883). The turning point of the analysis on product differentiation was the model developed by Hotelling (Hotelling, 1929). He proposed the first computed address branch model. The basic elements of this model came from the fundamental works of Christaller (Christaller, 1966) and Losch (Losch, 1938).

The greatest intuition of Hotelling was to introduce the Bertrand model in a differentiated oligopoly. He showed that when two firms sell differentiated product or have a different geographic locations, price competition leave the price high enough to cover capital cost and then reach a stable long run equilibrium. The assumption of Hotelling’s model are several but the principal was the one-dimension of the characteristics space. This assumption is one of the most important element which led to consider product differentiation as a way to pre-empt the market and avoid the entrance of new competitors.

The other source of success of address models came from the development of theories on the demand function for differentiated products. These models were perfectly adapted to the address concept. In this regard, one of the most important reference is the Lancaster’s model in which consumers’ preferences are defined over a bundle of characteristics which themselves are embodied in goods (Lancaster, 1966).

How the historical perspective shown the main difference between the non-address and address model is the structure of preferences. In the former the preferences are symmetric whilst in the latter are asymmetric. Thus, reference models of product differentiation are the Chamberlin’s monopolistic competition and the Hotelling address model.

2.2. Demand for differentiated products

The main reference in economic literature to investigate the demand for differentiated goods is the characteristic approach developed by Lancaster (Lancaster, 1966).

This theoretical contribute considers products as a bundle of several characteristics, hence consumers will choose the good which maximize their utility, and since the preference are asymmetric each consumer will buy a different kind of good\(^2\).

\(^2\) The basic assumptions of the Lancaster’s model are:
- Double the consumption of products yields double the consumption of characteristics;
- Characteristics from different products can be added together;

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The first step is to define the utility function, the simplest form is with the quantities of consumed characteristics as independent variable. In case of only two characteristics the utility function could be expressed by the following relation.

\[ U^i = U^i(c_1, c_2) \]  

(2.1)

The marginal utility of each characteristic is positive then both are desirable. An example here might be ‘breakfast cereals where a bulky breakfast cereal satisfy appetite can be mixed with a tasty breakfast cereal satisfying the need for an attractively presented meal’ (Ireland, 1987).

The functions of such characteristics is configurable how:

\[ c_1 = a_{11}q_1 + a_{12}q_2 \]  

(2.2)

\[ c_2 = a_{21}q_1 + a_{22}q_2 \]  

(2.3)

Where \( q_1 \) and \( q_2 \) represent the amounts of consumed product, whilst \( (a_{11}, a_{21}) \); \( (a_{12}, a_{22}) \) are quantities of two characteristics which are obtained per unit consumption of the first and second products respectively. The functions of \( c_1 \) and \( c_2 \) are called consumption technology.

If only good 1 is bought we are in the point \( R_1 \), this represents the amounts of characteristics that can be consumed, while \( R_2 \) represents the amounts if only product 2 is bought. The line \( R_1R_2 \) represents the effect of purchasing a convex combination of the two commodities. A point inside the area \( OR_1R_2 \) but not on the line \( R_1R_2 \), although feasible, implies an expenditure of less than the budget constraint.

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- Commodities are completely divisible so that the first and second condition above refer to consuming fractions of units of different products as well as number of units;
- To describe the products are sufficient just two characteristics.
2.3. *Imperfect competition models and geographical spaces*

This section describes the well-known Hotelling model (Hotelling, 1929). This address model is very important because it illustrates the market with differentiated products in a very real way. Furthermore, the Hotelling’s contribute is relevant because it is useful to analyze the entry process in horizontal differentiated market. The most important basic assumption is the concept of asymmetric preferences, which means that if consumer’s preferred brand is $x$ then consumer prefers brands that are close to $x$ in terms of their characteristics. One of the implications of the asymmetric preferences choice is that competition among firms is localized, so each movement of one firm has an effect on the behavior of other firms in the market.

As Hotelling argued, in a differentiated market firms compete on price, so they engage a Bertrand competition. Naturally, the differentiation avoids that the outcome is the same in the original Bertrand model, because firms can exploit market power. This means that firms can charge price above marginal cost, but not at monopoly level because products are slightly substitutes.

In the Hotelling’s model there are two firms, which are free to choose the location in the Main street, a unit line of $X$ length. The products are homogeneous but for consumers they are differentiated in two characteristics, on the geographical location and on transport cost necessary to reach the
Consumers are uniformly distributed along the line and each of them buy just one unit of the good. Customers will buy from the firm that will charge the lowest delivered price. The price for consumers is composed of the price of the good plus the transportation cost to reach the seller. An interesting way to analyze this model is the interpretation of Gabszewicz and Thisse (Gabszewicz et al., 1986). They showed how, with the assumption of quadratic transportation costs, it is possible to find a Nash equilibrium on price and location. The representation of the uniformity distribution in the main street is drawn in Figure 2.2.

Each firm is symmetrically located at a distance \( e \) from the mid-point. There is \( s_1 = \frac{1}{2} - e \) and \( s_2 = \frac{1}{2} + e \). The whole price is equal to the price of the good \( (p_i) \), and the transportation cost \( t(s,s') \). Then \( t(s,s') = c \cdot |s-s'| + e (s-s')^2 \); \( c, e \geq 0 \).

The starting point is to locate the indifferent consumer, which is the one for whom buying from firm 1 or firm 2 is equivalent.

\[
p_1 + t(s_1, \chi) = p_2 + t(s_2-\chi)
\]  (2.4)

Gabszewicz and Thisse considered in their model the price of firm 2 as given at \( p_2^* \). They built the demand curve for firm as:

\[
\chi_1(p_1, p_2^*) = 0 \text{ if } p_1 > p_1^{++}
\]

\[
= \frac{p_2^* - p_1 + 2ac}{4ae} \quad \text{if } p_1^{++} > p_1 > p_1^{-} \quad \text{which is equal to 1 if } p_1^{-} > p_1 \geq 0
\]

The derivation of the demand curve opens the door to several special cases, for instance, if \( e = 0 \), we have \( p_1^{++} = p_1^{-} = p_2^* = p_1^* = p_1^{--} \). In this case if \( p_1 > p_2^* \Rightarrow \chi_1 = 0 \) and if \( p_1 < p_2^* \Rightarrow \chi_1 = 1 \), the ultimate result in this case is that when firms chose to locate at centre of the main street, the only equilibrium is the Bertrand result \( p_1^e = p_2^e = 0 \) (Beath et al., 1991). The

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3 The geographical distance could be used also to measure the distance in consumer’s tastes.

4 In economic terms, central location for both firms means that the products are homogeneous therefore perfect substitutes. In this case, firms have no market power so the outcome is like in the Bertrand model.
relevant elements for the shape of the demand curve are the transportation costs. These have influence on the continuity and concavity of demand and profits functions.

Gabszewicz and Thisse found that if $e$ is large enough, a Nash equilibrium in prices will exist. This is the case when a boundary $\chi$, is between the two firms. Particularly, the necessary condition is

$$2a \geq \min \left[ \frac{1}{3}, \frac{\sqrt{c}}{2\sqrt{e} + \sqrt{c}} \right]$$

(2.5)

The result found by Hotelling was different because he developed the principle of minimum differentiation. He discovered that firms choose to locate close to each other, in a way to conquer the biggest part of the market. Then, both firms choose to locate in the middle of the main street, they choose minimum differentiation. The incentive for the firms to locate close to each other is known as agglomeration effect. Nevertheless, in real markets, it is possible to see how firms do not prefer to locate close to each other but they have the incentive to locate far from the others. Furthermore, if firms decide a location more distant from the others, they have market power to exploit. In other words, firms can charge a price above marginal cost thereby making positive profits. This principle is known as strategic effect. It is possible to see how this effect is completely absent in Hotelling’s address model. The relevant question to find the Nash equilibrium in differentiated market is to see which of these two forces is dominant. For this reason, several scholars tried to remove some of the basic assumptions made by Hotelling.

3. Entry process in markets with differentiated products

3.1. Pure profits and free-entry condition

This section analyzes the model developed by Eaton and Lipsey to start the study the entry process in differentiated markets (Eaton and Lipsey, 1978). Consider the Hotelling’s main street in which there are two firms (Firm 1 and Firm 2) located at a distance $d$ from each other and consumer are uniformly distributed over its length. The costs are represented by $c$, transportation cost, $m$ marginal production cost and $F$ as fixed cost. The aim is to see the price and location decisions of a newcomer (Firm 3), which wants locate in the industry between two existing firms. The important assumption here is that the location of existing firms is fixed.

Eaton and Lipsey defined $V = g(p,v,x)$ indicating that the price that any firm expects a neighbouring firm to charge depend on its own price ($p$), the
price currently being charged by the neighbouring firm \((v)\), and the distance between the two firms \((x)\).

Firm 3 wants to locate at point \(k\). Let point \(k'\) be the point at which it anticipates that its delivered price will be equal the one of the Firm 2. Then it is anticipated that at \(k'\) the following condition will hold:

\[
p + c(k'-k) = g(p, v,d - k) + c(s-k')
\]  

(3.1)

solving for \(k'\):

\[
k' = \frac{1}{2c}[g(p,v,d-k) + c(d+k)-p]
\]  

(3.2)

if substitute the (2.2) into the right side of (2.1) it is possible to find how the anticipated delivered price at \(k'\) is:

\[
P_2 = \frac{1}{2}[g(p, v, d-k) + c(d-k)]
\]  

(3.3)

as a result the demand function of Firm 3 between \(k\) and \(k'\) is:

\[
\int_{k}^{k'} \int f[p + c(x - k)]dx
\]  

(3.4)

Considering the demand function of an individual consumer located at a distance \(x\) from the origin as a function, \(f\), of the delivered price.

At the end, after found the anticipated delivered price of Firm 1 and thus the expected demand to the left of \(k\), is possible to show the demand function for Firm 3:

\[
Q(k, p) = \frac{1}{c} [F(P) + F(P) - 2F(p)]
\]  

(3.5)

Firm 3 wants to maximize his profits then:

\[
\frac{\partial Q}{\partial k} = 0 \quad \text{and} \quad \frac{\partial^2 Q}{\partial^2 k} < 0
\]  

(3.6)

The optimal location is when \(k\) is equal to zero thus when Firm 3 will choose to locate in the middle of the main street. Firm 3 is able to anticipate that result, so through (3.1) it is possible to see how when \(k = 0\) we have

\[
P_1 = P_2 = P = \frac{1}{2} [p + g(p,v,d) + cd]
\]  

(3.7)
From (3.5) we can obtain the expected demand function when Firm 3 choose the middle of the line:

$$Q(0, p) = \frac{2}{c} [F(P) - F(p)]$$  \hspace{1cm} (3.8)

The relative profits are:

$$\pi = (p - m)Q - F$$  \hspace{1cm} (3.9)

Through the maximisation condition, Eaton and Lipsey found how exists at least one value of $p, p^*$ that satisfy the maximisation problem.

From this model we can draw some interesting conclusions. First of all, if every firm choose a price ($p^*$) and location at distance $d$ from the others, no firm wishes to move or modify its price. This result is called the fixed-numbers equilibrium. Secondly, the maximizing price is independent from the fixed cost $F$. Therefore, in this case firms in the markets choose the location and the price that maximize their profits, but it is still necessary to see if in the case of entrance of Firm 3 these profits remain positive.

Eaton and Lipsey shown how in case of entry the demand of the newcomer lies always to the left of the demand curve that existing firms face. Thus, even if the incumbent firm earns positive profits, the anticipated demand curve of the entrance will lies below its average cost curve, so entry is blocked.

The result found by Eaton and Lipsey reveal how incumbents can earn positive profits without attracting entry by newcomers. From this result was developed several models with the aim to explain this circumstance.

3.2. Entry deterrence strategies in markets with differentiated products

The most important model on deterrence strategy in markets with differentiated products was developed by Schmalensee (1978). This theoretical treatise is crucial to understand why product differentiation is considered an entry barrier.

Schmalensee considered product proliferation as a sunk cost, therefore incumbents erect an entry barrier through filling up the characteristics space. The economic consequence of this statement is the condition of immobility for the established firms. With sunk cost necessary to enter in the market and localized competition an entry barrier is created. Then, firms can choose to pre-empt the market in a way to leave not enough space for the entrance of other firms.
One of the central assumptions is about the shape of the characteristics space, instead of a linear segment, Schmalensee considers a unit circular market.

There are \( n \) firms in the market, each produce a single brand and are located at a distance of \( 1/n \) around the circular market. The demand function is decreasing in price and in the number of firms into the market, and then could take the following form:

\[
Q(p, n) = a(p)b(n); \quad \text{with} \quad \frac{\partial a}{\partial p} < 0 \quad \text{and} \quad \frac{\partial b}{\partial n} < 0
\]

Fixed costs and marginal costs are respectively \( F \) and \( m \), therefore, the profit function is:

\[
\pi(p, n) = (p - m)Q - F = A(p)b(n) - F
\]

\[
\text{with} \quad A(p) = (a(p)(p-m))
\]

Consider a given price \( p \), \( \tilde{n} \) is the solution to the equation \( \pi(p, n) = 0 \), in this case profits are positive only when \( n \leq \tilde{n} \).

The newcomer wish to localize in the middle between two existing firms, hence when other firms charge a price \( p \), the profits are \( \pi(p, 2n) \). This situation arise because the distance between the newcomer and one of each incumbents is half of that between existing firms.

For as long as \( \tilde{n}/2 < n < \tilde{n} \) all existing firms can earn positive profits\(^5\), but for newcomers entrance is not a dominant strategy because they will earn a profit equal to \( \pi(p, 2n) \) and since \( 2n > \tilde{n} \) entry is not profitable. Hence, also with this model the result found by Eaton and Lipsey (Eaton and Lipsey, 1989) is confirmed, established firms are able to earn positive profits even with free entry condition. This also means that product proliferation could be a strategy to avoid the entrance in the market. Existing firms can choose to produce different varieties of the same product in a way to conquer the necessary space to avoid the entrance of newcomers. That is like if there are different firms, so the profits for each variety decrease with the number of varieties produced, but the whole profits for the incumbent firms do not decrease because it is the producer of all varieties into the market. For instance, suppose that \( \tilde{n} \) is equal to ten and there are two incumbent firms.

\(^5\) All firms earn positive profits because since \( n < \tilde{n} \) and \( \pi \) is decreasing in \( n \) if \( \tilde{n}/2 < n < \tilde{n} \) \( \Rightarrow \pi(p, n) > 0 \). Whilst for newcomers we have that the expected profits is too low because the number of firms that maximize the profit function is to high \( 2n > \tilde{n} \).
They can choose to produce only two products, but in this way there is space for the entrance of eight newcomers. Now, suppose that each incumbent starts to introduce two varieties of its product. The profits for each varieties are lower than the previous case because the profit function is decreasing in the number of firms or varieties in the market, but the whole profits for each incumbent is the same because he is the only producer of two varieties\textsuperscript{6}. Hence, existing firms can find profitable to fulfill the market through the production of five varieties each. In this case, the profits for each incumbent firm are the same but the entrance is blocked.

Schamalensee demonstrated how product proliferation is the incumbent dominant strategy, because it avoids the entrance of new competitors and at the same time earns positive profits under free entry conditions. Finally, brand proliferation could be defined as the production of the minimum number of brands that deters entry. In the following of this work we tried to remove some basic assumptions of these models. The results are significantly interesting because the outcome is completely different.

4. Markets with horizontal differentiated products: Is there space enough?

As shown in the previous section, horizontal product differentiation could be considered as an entry barrier. This conclusion has an important result in antitrust investigation\textsuperscript{7}.

The endeavor of this section is to show how product differentiation could not be an entry barrier. In every market it is possible to find firms selling differentiated goods or produce several varieties of the same item. Considering horizontal differentiation as an entry barrier means that the true role played by differentiation is not fully understood. In several differentiated markets, it is possible to see the entrance of newcomers\textsuperscript{8}. Therefore, a deeper view in the real world tells us that incumbent firms cannot avoid the entrance of new competitors only through the brand proliferation strategy.

\textsuperscript{6}This is the case when marginal cost to produce another variety is zero.

\textsuperscript{7}Considering product differentiation as an entry barrier has deep implications on the firm discipline. Indeed, firms which produce several varieties of the same good could be declared to be in a dominant position. Therefore, numerous behaviors could be considered as abuse of dominant position under the article 82 of the Treaty or a merger could be blocked because the dominant position (Regulation 139/2004).

\textsuperscript{8}For instance could be considered the case of cigarettes, where there is an incumbent which produce several varieties of the same good, and at the same time there are newcomers trying to enter the market with cheaper cigarettes (Philip Morris with Marlboro vs. Pall Mall). Otherwise, we can consider the car sector and the case of Autogerma which produces numerous kinds of cars in the B segment (Audi A3, Seat Ibiza, Volkswagen Golf and Skoda Fabia), but at the same time we saw a newcomer in the segment enter the market (Bmw with the Series 1).
A crucial role, in horizontal differentiated markets is played by consumers’ preferences. Products are differentiated in characteristics that become goods subjectively differentiated. Hence, the preferences for each variety depend on consumers’ tastes. As shown in the previous section, consumers buy differentiated products on the basis of the characteristics of the goods. Then, they consider products as a bundle of characteristics. In this view, the multidimensional analysis is the best approximation of the characteristics space.

4.1 One dimensional vs. multidimensionality characteristics space

This section removes some basic assumptions of the Hotelling’s model. First of all, the dimension of the characteristics space. Therefore, it considers a multi-dimensional characteristics space in a way that consumers’ preferences can be better approximated.

The intuition of multidimensionality manages to meet consumers’ tastes with more precision, with powerful mathematical tools, it is possible to consider this hypothesis. Besides, most analysis of product differentiated markets are based on the one-dimensional characteristics space assumption. Clearly, it was made for mathematical convenience.

As described in the previous parts, under all basic postulations of a classical version of the Hotelling’s model there is not a uniqueness of price equilibrium. Removing some of these assumptions it is possible to find the Nash equilibrium in price and location (Gabszewicz, 1986). This result is reached with the supposition of quadratic transportation cost. Furthermore, another principle that does not seem to be always present in differentiated market is the minimum differentiation principle. On the basis of this statement elaborated directly by Hotelling, firms will choose to locate close to each other with a consequent Bertrand competition. D’Aspremont et al. showed how firms want to maximize product differentiation to relax price competition and exploit market power (d’Aspremont et al., 1979).

Several scholars focused their analysis on the multi-dimensionality of the characteristics space, with two (Economides, 1986) or three dimensions (Ansari et al., 1996).

Assume the presence of two firms and the product variants are given by the location of the firms in the space $\mathbb{R}^n$. Then the location of firm A is given by a vector $a = (a_1, \ldots, a_n)$ and the location of firm B is described by a vector $b = (b_1, \ldots, b_n)$.

The hypothesis of uniformity distribution of consumers in the characteristics space is maintained, specifically consumers are distributed over the unit hypercube $C = [0,1]^n$. If we consider the continuous and non-negative density function $g(z)$ with the location of a consumer described by

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the vector \( z = (z_1, \ldots, z_n) \), it is possible to find the total population as
\[
\int_{\mathbb{R}^n} g(z) dz = N.
\]

Through some mathematical tools, it is possible to describe analytically the utility function in a multidimensional characteristics space; to do that, we can use the Euclidean distance as a measure of the distance between two points in the space.

Consider \( V_i(z) \) as utility function with \( i = A, B \); a consumer buying good A enjoys a utility equal to the surplus that derives from consuming the good, minus the cost to buy it (the price) and minus the distance from his ideal good (Caplin et al., 1991).

\[
V_A(z) = S - p_A - \sum_{k=1}^n \ell_k(z_k - a_k)^2
\]

(4.1)

S is equal to the surplus that the consumer enjoys by consuming the good, \( p_A \) represents the price paid to buy the good and the last term is the square of the weighed Euclidean distance between the consumer’s ideal point and the location of variant A; where \( \ell_k \) represents the coefficient of characteristic \( k \).

After the expression of the utility function, it is possible to derive the demand curve for firm A. Assuming that consumers have unit demand and that S is large enough for all consumers to buy at any location with the corresponding price; the demand for variant A is then defined by the group of consumers for whom this variant is preferred to variant B:

\[
\{ z : V_A(z) \geq V_B(z) \}
\]

(4.2)

To find the Nash equilibrium in price and location we referred to the model developed by Caplin and Nalebuff. They found the conditions for the existence and uniqueness of price equilibrium. This result depends on the functional form and the distribution of consumer preference and the uniform distribution complies with \( \rho \) – concavity (Caplin et al., 1991). This condition with a twice differentiable demand curve and the log-concave uniform distribution, implies the uniqueness of price equilibrium for each location pair.

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9 For simplicity consider the characteristics weighed equally across consumers.

10 It is assumed that each variety is produced at constant marginal cost equal to zero.
4.2. Max-min principle, multi-characteristics space and consumers preferences

This section describes the work developed by Irmen and Thisse to show how it is possible to find the product positioning in multidimensional characteristics space (Irmen and Thisse, 1996).

The power of the multidimensionality shape of the space lets us to draw some interesting conclusions. With \textit{n}-dimensions, it is possible that the agglomeration and the strategic effects can live together. Furthermore, firms can choose minimum differentiation along 1\textit{n}-characteristic and maximum differentiation along the other \textit{n}-1 characteristics.

Besides, Irmen and Thisse have shown how, in a product differentiated duopoly, it is possible to find the Nash equilibrium in price and location. In this case firms choose maximum differentiation along \textit{n}-1 characteristics and minimum along the \textit{n}\textsuperscript{th} characteristic\textsuperscript{11}.

4.2.1. The demand function in a multidimensional space

First of all it is necessary to estimate the demand function for the firms in the market. Hence, it is necessary to localize the indifferent consumer, who can buy variety A or B indifferently.

The indifference condition is:

\[
p_A + \sum_{k=1}^{n} t_k (z_k - a_k) = p_B + \sum_{k=1}^{n} t_k (z_k - b_k) \tag{4.3}
\]

After that the relative consumer address is equal to:

\[
\hat{z}(z_1, z_2, \ldots, z_{n-1}) = \frac{p_B - p_A + \sum_{k=1}^{n} t_k (b_k - a_k)}{2t_n(b_n - a_n)} - \sum_{k=1}^{n} \frac{t_k (b_k - a_k)}{t_n(b_n - a_n)} z_k \tag{4.4}
\]

\textsuperscript{11} The same results are found by A. Ansari et al, (1996) and by N. Economides (1986).
With the assumption of \( b \geq a \), the slope of the hyperplane is negative along each dimension. Figure 4.1 provides an illustration when \( n = 3 \). The size of the market place of each firm is equal to the sum of the volume of pyramids. The dimensions of the flanks of these pyramids depend on the price differentials. Therefore, the marginal consumer will choose a determined variety on the base of the price differential and the distance of each variety from his ideal point.

Irmen and Thisse assume that

\[
\sum a_i \leq 1, \quad \sum b_i \leq 1, \quad \sum c_i \leq 1
\]

and define the \( n \)th characteristic as dominant, whilst the others are dominated (Irmen et al., 1996). Furthermore, through several passages they found how, under condition of strong dominance of the \( n \)th characteristic, demand middle piece for firm A is equal to:\(^{13}\):

\[
\tilde{D}_{n-1} = \frac{\tilde{p}_A - \tilde{p}_B + \sum_{i=1}^{n} t_i (b_i - a_i) - \sum_{i=1}^{n-1} t_i (b_i - a_i)}{2t_n (b_n - a_n)}
\] (4.5)

This means that firm A can choose a pair of location and price in this demand area.

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12 A very clear analysis with two characteristics was made by A. Ansari et al (1996).
13 The middle piece of firm B is determined since \( DB=1-DA \).
4.2.2. Nash equilibrium in price under the max-min principle

Through demand function in (4.5), it is possible to solve the first order condition of maximization to find a unique solution:

\[
\tilde{p}_A = \frac{2t^*_t (b_n - a_n) - \sum_{k=1}^{n} t_k (b^2_k - a^2_k) + \sum_{k=1}^{n} t_k (b_k - a_k)}{3} \quad (4.6)
\]

\[
\tilde{p}_B = \frac{4t^*_t (b_n - a_n) - \sum_{k=1}^{n} t_k (b^2_k - a^2_k) + \sum_{k=1}^{n} t_k (b_k - a_k)}{3} \quad (4.7)
\]

This outcome is interpretable through the principles of minimum differentiation found by Hotelling and the principle of maximum differentiation found by d’Aspremont et al (d’Aspremont et al., 1979). Moreover, firms, on one hand, have the trend to locate close to each other in such a way as to exploit bigger market shares, but with high price competition. On the other hand, firms have the incentive to locate far from other firms to relax price competition but with a reduced market place. In one dimensional characteristics space, these two effects were a trade off. In a multidimensional space these two principles can coexist together. Firms will choose minimum differentiation in \(n-1\) dimensions and maximum differentiation along the \(n^{th}\) characteristic. The resulting price is not the Bertrand one, because firms can still differentiate their products. Hence, in this way the price is not at monopoly level neither at a competitive level, but in a point between these two values.

In our terms, it is possible to say that if a newcomer wants to enter into the market with differentiated products can apply the max-min principle. The final result is that there is always space enough for entrance and production of \(n\)’s different varieties. In this way, newcomers can sell products very close to the ones produced by the incumbent, and then earn sufficient profits to survive in the market.

Irmen and Thisse, in their work found also the location equilibrium for both firms. The result for \(n=3\) is that maximum differentiation along all three characteristics is never an equilibrium. They gave an evidence how also maximum differentiation in two of three characteristics is never location equilibrium\(^{14}\).

\(^{14}\) Also in two dimensional characteristics space is confirmed the principle of the max-min principle: See T. Tabucchi (1994), pp. 207-227.
5. Competition in horizontal differentiated markets

The aim of this section is to demonstrate how horizontal product differentiation could not be an entry barrier. Through game theory tools and the well known folk theorems it is possible to show how for incumbent and newcomer it is better to cooperate and, then choose the duopoly solution instead to start a war price. In this regard, brand proliferation is not always the best strategy for the incumbent, and for newcomers it is not always logic to enter the market at whatever cost. In other words there is a situation in which the best firms can do is to collude and choose to avoid the pre-emption strategy and not enter in a hostile way. In real markets this concept seems very clear. Indeed, the entrance of newcomers in differentiated market is observable everyday and in every sector.\(^{15}\)

The max-min principle with price competition permits to explain how newcomers can enter the market, and solves the trade-off present in one dimensional characteristics space between agglomeration and strategic effect. Furthermore, a newcomer can conquer new consumers through decreasing its price; this result was confirmed also by Ansari et al. (Ansari et al, 1996). The interesting conclusion is that newcomers can profitably enter the market. Throughout the production of varieties near the incumbent’s ones and a price reduction, new entrants can conquer a sufficient market share. This market share is potentially composed by three groups of consumers. The first group is composed by consumers in the most elastic part of the incumbent’s demand curve. In other words, all customers for whom the price is more important than the variety. The second group could be arranged by all consumers for whom the new varieties produced by newcomers are preferred to the incumbent’s variety. Finally, the last group could be composed by new consumers who before did not buy the good.

After underlining how in a multidimensional environment there is space enough for entrance we can study the entry process under the hand lens of game theory. Indeed, we can read the entry process in a differentiated market as a classical prisoner dilemma. The two prisoners could be seen as the incumbent in the differentiated market and the newcomer which is trying to enter.

As well known in this kind of game the outcome is no collaboration, in our terms firms start a battle, where the incumbent wants to avoid the entrance, and the newcomer wishes to enter the market. No collaboration means that the incumbent starts to crowd the characteristics space through brand proliferation; the newcomer try to enter the market selling products at a price substantially lower than the one charged by the incumbent.

\(^{15}\) For an application of the max-min principle with price competition to the cigarette market and the automotive sector see M. Di Cola (2005).
Therefore, the assumptions at the base of our model are:

1. **Entrance**: Newcomers can enter the market following the max-min principle and a price reduction. The incumbent can pre-empt the market in a way to obstacle the entrance and become it costly;

2. **Players and market structure**: There are two firms at each stage. Thus, in different stages the newcomer could change. In the first stage only the incumbent is present in to the market, thus the structure is a monopoly with horizontally differentiated products. Eventually, after the entrance market structure could become a duopoly;

3. **Strategies**: Firms can choose to collaborate or not.
   
a. **Collaborate**: From the incumbent viewpoint, it decides to avoid the proliferation strategy, and at the same time it saves the relative costs. From the new entrant point of view, it could choose to collaborate, and then avoid to enter the market aggressively. In other words it can enter in the market at price slightly lower than the one charged by the incumbent without attracting all consumers\textsuperscript{16}.
   
b. **Not collaborate**: incumbent avoids collaboration through the pre-emption of the characteristics space. It is important to underline for the incumbent firms proliferation it is not costless. These costs are low but not zero, because the firm is already producing the good, then it has only to change the variety. Therefore, we assume that the incumbent must bear an increase only in variable costs, and not in fixed ones. Newcomer could not collaborate through an aggressive and hostile entry. It can enter the market at a very lower price (under its average costs) than the one charged by the incumbent. In this regard, we assume that new entrant charges a price which is not sufficient to recover the cost, and then not sufficient to stay in the market. The aim of this strategy is to cause as much losses as possible to the incumbent, thus it represents a threat\textsuperscript{17};

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\textsuperscript{16}This hypothesis is coherent with the market structure (assumption 1), indeed firms operate in a non competitive environment. Therefore, a firm charging a price slightly lower than its competitor does not conquer the whole market.

\textsuperscript{17}At first sight this threat could be not credible, but in the following we consider the possibility for the newcomer to get credit.
4. **Stages and sub games:** the game is infinitely repeated (*folk theorem*), we assume that in each stage there could be a new entrants who wants to entry. Each firm state a threat, which in case of no collaboration could be applied in the next stage until the betray firm restart to collaborate (*trigger strategy*). The threat of the incumbent is about the proliferation strategy, whilst the threat stated by the newcomer is relative to an aggressive entrance or behavior;

5. **Profits:**

   a. monopoly profits even less the cost of producing another variety (in case of brand proliferation) are bigger than duopoly profits. The cost to pre-empt the market is less than the loss of half market in case of duopoly ($\pi^M - nc > \pi^D$);
   
   b. when the newcomer stays out profits are equal to zero, we called outside profits. If the newcomer enters the market aggressively, it is necessary to get credit, the cost of the loan is $ng$.

In strategic form the game could be represented as in Figure 5.1. Now we proceed following a step-by-step analysis. We consider several cases of the game (one stage, two stages and infinitely repeated game) only to explore in detailed the mechanisms of the entry process. However, our conclusions are valid only if all the assumptions are met, particularly only if the game is infinitely repeated.

**Fig. 4 – Sub game (g) in differentiated products**

<table>
<thead>
<tr>
<th>INCUMBENT</th>
<th>Collaborate</th>
<th>Not</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>NEWCOMER</strong></td>
<td>Collaborate</td>
<td>$\pi_1^D; \pi_2^D$</td>
</tr>
<tr>
<td></td>
<td>Not</td>
<td>$\pi_1^D; \pi_2^D - ng$</td>
</tr>
</tbody>
</table>

**Step 1 – One stage game**

Where the first value represents the payoff for the incumbent (Firm 1), whilst the second value indicates the newcomer (Firm 2). In the first cell it is represented the outcome collaborate- collaborate. Both firms gain duopoly profits without bear any strategic expense. In the second cell (first row, second column) the incumbent decides to not collaborate and the newcomer
chooses collaborate. Thus, Firm 1 earn monopoly profits, because the proliferation strategy, but it must bear the relative costs ($nc$); Firm 2 gain outside profits (null profits) because stay out without expenses. In the third cell (second row and first column) it is represented the outcome collaborate-not collaborate. Firm 1 does not pre-empt the characteristics space, than it earns duopoly profits. Firm 2 decides to not collaborate, and than enter the market in a hostile way, it charges a price below its average cost causing potential losses for the incumbent. In the fourth cell (second row and second column) it is shown the worst outcome, where both firms decided to not collaborate. In that situation Firm 1 bring in monopoly profits because the proliferation strategy bearing relative costs; whilst Firm 2 get outside profits minus the cost of getting credit ($ng$).

**Step 2 – two stage game**

What happen if we consider a two stage game? In this case the outcome is not assumed as in the previous example. In the one stage game the newcomer cannot enter the market at a lower price because it has not the resources to bear this effort. In a two stage game we can assume that the new entrant takes a loan to endure the expenses in the first stage and recover them in the next ones where it acts as duopolist.

In considering this option it is necessary to analyze the cooperation outcome. Cooperation means that the incumbent avoid the proliferation strategy and the newcomer enters the market charging a price lower until a level which guarantee to survive in the market. In this case, incumbent and newcomer play a duopoly game which is profitable for both; the former saved the cost of brand proliferation and the losses in case of high competition, the latter put away the cost to get credit.

**Step 3 – Infinitely repeated game**

At this point we can improve our game introducing the hypothesis of infinitely repeated game with perfect information. This hypothesis is coherent with reality, because instead to consider just one newcomer we can assume that there is always a newcomer at each stage of the game which is trying to enter the market. In this situation, the pre-emption strategy is not feasible for an infinite number of cases. Consequently, for both firms it is convenient to cooperate and act as duopolists.

Formally we have a game $G = \{A_1, ..., A_n; u_1, ..., u_n\}$, with a player set $I = \{1, ..., n\}$, the action profile is $A = X_{i=1}^n A_i$. Each player has a von Neumann-Morgenstern utility function. A repeated strategy $s_i \in S_i$ for player $i$ is a

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18 For a deeper analysis on the nomenclature used see R. Gibbons (1992) and J. Ratliff (1996).
sequence \( s_i = (s_i^0, s_i^1, \ldots) \), where each \( s_i^t \) is history dependent stage game strategy. The history at time \( t \), \( h' \in A' \) is the sequence of action profiles \( h' = (a^0, a^1, \ldots a^{t-1}) \).

With a discount factor \( \delta \in (0,1) \) at which players discount their future payoffs, the average discount payoff for an infinitely repeated game is \( A(v_i^\delta, \delta) = (1 - \delta) \sum_{i=0}^{\infty} \delta^{i-1} \pi_i' \). Thus, the infinite sum for the whole game is

\[
\sum_{t=0}^{\infty} \delta^t = \frac{\delta^T - \delta^{T+1}}{1 - \delta}
\]

In this kind of games for each \( t < T \) the outcome in the sub game \( t \) it is not a Nash equilibrium. This result was proved by Friedman through the development of one of the first Folk theorem (Friedman, 1971). Thus, in our case the two firms can cooperate at each stage of the game even if the only possible equilibrium for the game \( G \) is not collaborate. This outcome is possible because the threat, which is called trigger strategy.

A further confirmation of this result come from Fudenberg and Maskin. They demonstrated how in games with two players and payoffs \( r_1 \) and \( r_2 \) could substitute the equilibrium payoffs \( e_1 \) and \( e_2 \) in the Friedman theorem (Fudenberg and Maskin, 1986). In our terms this theorem means that even if for the incumbent it would be logic to avoid the entrance, and for the newcomer it would be natural to entry at whatever cost, even selling goods below the average cost, the final outcome in infinitely repeated games they can collude and act as duopolists. Therefore, even if collaborate-collaborate is not a Nash equilibrium in pure strategy in each sub game it could be the solution for the whole game, and than the final outcome.

6. Conclusions

Everyday consumers face the difficult choice between products which are differentiated in one or more characteristics. Particularly, horizontal differentiated products are sold in every market. The huge diffusion of this phenomenon calls for a deeper analysis on its effects in the market. Indeed, effects coming from product differentiation are several, more of these positive for consumers and firms. Usually, negative effects are not directly consequences of product differentiation, but are ancillary effects linked to it. We are referring at advertising, brand loyalty and goodwill which become the production of several varieties of the same good a barrier to entry.

In several markets there is the possibility to see the entry of newcomers in differentiated markets. Therefore, the point is to discover how firms can enter in these kind of markets. Naturally, this is a more difficult task than in

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situations with homogeneous goods, but entry is not impossible. The awareness of the possibility to entry for newcomers led us to investigate on the mechanisms of this process. Particularly, we started to remove some basic assumptions from the classical models on horizontal product differentiation. The first one is the difference between geographical and artificial spaces, consumer preferences are considered in the second category. Secondly, we introduced the multi dimensions of the characteristics space, which is linked to the powerful max-min principle.

Finally, we interpreted the entry process as an infinitely repeated game between the incumbent and newcomers. In this regard, using folk theorems we discovered how firms can choose to collaborate. Incumbent allow the entrance and new entrant charges a price slightly lower than the incumbent one. From this viewpoint, new entrant cannot gain the whole market because the market structure (differentiated products, asymmetric information etc.).

In this light, we can consider a possibility to entry, and explain how firms can choose at the same time the strategic and agglomeration effect.
REFERENCES


