ECONOMIC CAPITAL MANAGEMENT FOR INSURANCE COMPANIES USING CONDITIONAL VALUE AT RISK AND A COPULA APPROACH

Rossella Bisignani¹, Giovanni Masala², Marco Micocci³

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1. Introduction

The loss ratio (LR) for insurance companies is defined as the ratio of incurred claims and earned premiums for a specified class of insurance (Col). The company may estimate then its capital requirement for that particular Col by using Value at Risk (VaR) or conditional VaR (CVaR) of the loss ratio distribution at a specified probability value. The overall objective of the company is to evaluate the aggregate capital requirement through a weighted sum of marginal capital requirements for all the classes of insurance. Nevertheless, this procedure may tend to over-estimate the aggregate capital requirement because it does not take into consideration the real dependence amongst the different classes of insurance. In other words, perfect dependence does not allow considering diversification effects. In this paper, we present a model which permits to take into consideration real correlations of the several Cols. Thanks to copula functions, we are able to generate (by Monte Carlo simulations) correlated loss ratios with known marginal distributions. This approach is described through a numerical example that used data collected from some of the most important Italian non life insurance companies. We show then the diversification benefit thus obtained. We conclude the paper building an efficient frontier on the plane LR – CVaR; the efficient frontier may be considered a useful tool to manage the global company risk.

In the following sections of this paper, we present a model for estimating the economic capital for non life insurance companies; as already said the

¹ PHD student in Financial and Actuarial Mathematics - University of Rome “La Sapienza” (rossella.bisignani@uniroma1.it).
² Researcher in Financial and Actuarial Mathematics - University of Cagliari (gb.masala@tiscali.it).
³ Full Professor of Financial and Actuarial Mathematics - University of Cagliari (micocci@unic.it)
model takes into account real dependences amongst losses coming from different classes of insurance (CoI).

The main purpose is determining the diversification benefit considering a real dependence structure through a copula approach.

Our analysis is based on the calculation of loss ratios (LR) that represent the ratio of incurred claims and earned premium for a specified CoI.

We collected publicly available data from Italian non life insurance companies for the period 1980-2004 for the following five classes of insurance:

1. Accident
2. Land vehicles
3. Motor vehicle liability
4. Fire and natural forces
5. Third party liability

Through a best fitting analysis, we determined for each class of insurance the probability distribution of its loss ratio.

For each CoI we quantify its economic capital using risk measures like Value at Risk (VaR) and Conditional Value at Risk (CVaR) at a confidence level of 95% and 99% (we remind that CVaR is a coherent risk measure, which is an important property for risk management purposes; see Rockafellar-Uryasev).

Traditionally, the aggregate capital charge is obtained by summing the capital charges for each class of insurance. Nevertheless, this approach does not take into consideration diversification and hence tends to overestimate the capital charge.

In this paper, we propose a copula-based model that permits to consider correlations.

The empirical correlations coming from rough data have been modified on the base of some coherency elements (indeed, we have very few data for estimating correctly empirical correlations).

In particular the empirical values of the correlation coefficients have been compared with their “logical” and intuitive levels.

The logical values of the correlation coefficients depend on the definition of the CoIs; a deep analysis of the technical structure of each class of insurance has permitted to individuate three qualitative levels of correlation amongst the several CoIs:

- High degree (with a correlation coefficient equal to 0.75)
- Medium degree (with a correlation coefficient equal to 0.45)
- Low degree (with a correlation coefficient equal to 0.20)
The empirical values of the correlation coefficients have been compared with their “logical” values.

In some cases this comparison has substantially confirmed the empirical value; in other cases the empirical correlation coefficients have been forced to more reasonable levels (the “logical” values).

The numerical example shows that real correlation model leads to a lower capital charge.

We conclude the analysis with the construction of an efficient frontier based on a CVaR constraint. The paper is structured as follows: section 2 overviews some basic results about copula functions; section 3 introduces loss ratios and risk measures; section 4 presents the numerical example and section 5 concludes.

2. Copula functions and dependence structures

Definition.

An \( n \)-dimensional copula is a multivariate distribution function \( C \), with margins uniformly distributed in \([0,1]\) that satisfies the following properties:

\( (i) \quad C : [0,1]^n \rightarrow [0,1]; \)

\( (ii) \quad C \) is grounded and \( n \)-increasing;

\( (iii) \quad C \) has margins \( C_i \) satisfying

\[
C_i(u) = C(1, \cdots, 1, u, 1, \cdots, 1) = u \quad \forall u \in [0,1] \quad (i = 1, \cdots, n) \quad (2.1)
\]

The most important result is the following, due to Sklar:

Theorem.

Let \( F \) be an \( n \)-dimensional distribution, with marginals \( F_i \). Then there exists an \( n \)-copula \( C \) such that \( F(x_1, \cdots, x_n) = C(F_1(x_1), \cdots, F_n(x_n)) \).

If the marginals \( F_i \) are continuous, then the copula \( C \) is unique. Besides, if the density function exists, it will be given by:

\[
f(x_1, \cdots, x_n) = c(F_1(x_1), \cdots, F_n(x_n)) \cdot \prod_{i=1}^{n} f_i(x_i) \quad (2.2)
\]
with:

\[
c(u_1, \ldots, u_n) = \frac{\partial^n C(u_1, \ldots, u_n)}{\partial u_1 \cdots \partial u_n} \tag{2.3}
\]

The previous representation is called canonical representation of the distribution. Sklar’s theorem is then a powerful tool to build \(n\)-dimensional distributions by using one-dimensional ones, which represent the marginals of the given distribution. Dependence amongst marginals is then characterized by the copula \(C\).

We describe now some important multivariate copulas.

The **Gaussian** copula. Its parameters are the correlation matrix \(R\).

The Gaussian copula is given by:

\[
C_R^{Ga}(u_1, \ldots, u_n) = \Phi_R^n(\Phi^{-1}(u_1), \ldots, \Phi^{-1}(u_n)) \tag{2.4}
\]

where \(\Phi_R^n\) denotes the standardized multivariate normal distribution with correlation matrix \(R\) and \(\Phi^{-1}\) is the inverse of the standard univariate normal distribution.

The density of the normal copula is:

\[
c_R^{Ga}(u_1, \ldots, u_n) = |R|^{\frac{1}{2}} \cdot \exp \left( -\frac{1}{2} \zeta^T \cdot (R^{-1} - I) \cdot \zeta \right) \tag{2.5}
\]

where \(\zeta = (\Phi^{-1}(u_1), \ldots, \Phi^{-1}(u_n))\).

Finally, we can generate random numbers from the Gaussian copula through the following algorithm:

- find the Cholesky decomposition \(A\) of the correlation matrix \(R\);
- simulate \(n\) independent random variates \(z = (z_1, \ldots, z_n)\) from the standard normal distribution;
- determine the vector \(x = A \times z\);
- determine the components \(u_i = \Phi(x_i)\) \(i = 1, \ldots, n\). The resultant vector is \((u_1, \ldots, u_n)^T \bowtie c_R^{Ga}(u_1, \ldots, u_n)\).

**The Student t-copulas.** Its parameters are the correlation matrix \(R\) and the degrees of freedom \(\nu\).

It is the copula of the multivariate Student’s \(t\)-distribution.
Let $X$ be a vector with a $n$–variate standardized Student’s $t$-distribution with $\nu$ degrees of freedom, and covariance matrix $\frac{\nu}{\nu - 2} \Sigma$ (for $\nu > 2$). It can be represented in the following way:

$$X := \frac{\sqrt{\nu}}{\sqrt{S}} Y$$

(2.6)

where $S \sim \chi^2_\nu$ (the chi-square distribution) and the random vector $Y \sim N_n(0, \Sigma)$ are independent.

The copula of vector $Y$ is the Student’s $t$-copula with $\nu$ degrees of freedom.

The analytical representation is the following:

$$C_{u,R}^n (u_1, \ldots, u_n) = t_{\nu,R}^n \left( t_{\nu}^{-1}(u_1), \ldots, t_{\nu}^{-1}(u_n) \right)$$

(2.7)

or equivalently:

$$C_{u,R}^n (u_1, \ldots, u_n) = \prod_{i=1}^{n} \frac{\Gamma \left( \frac{\nu + 1}{2} \right)}{\Gamma \left( \frac{\nu}{2} \right)} \left( 1 + \frac{1}{\nu} u_i^\top \cdot \Sigma^{-1} \cdot u_i \right)$$

(2.8)

where $R_{ij} = \frac{\Sigma_{ij}}{\sqrt{\Sigma_{ii} \cdot \Sigma_{jj}}}$ for $i, j \in \{1, \ldots, n\}$ are the correlations. We also indicate $t_{\nu,R}^n$ the multivariate cdf of the random vector $\sqrt{\Sigma} \cdot Y$, where the random variable $S \sim \chi^2_\nu$ and the random vector $Y$ are independent. Besides, $t_{\nu}$ (cdf of the standard univariate Student distribution) denotes the margins of $t_{\nu,R}^n$.

Finally, the $t$-copula has the following density:
where \( \omega_i = t_{\nu}^{-1}(u_i) \).

If we choose a Student’s \( t \)-copula, the degree of freedom \( \nu \) can be evaluated with a log likelihood estimator.

Having chosen the marginals, this can be accomplished through a Maximum Likelihood Estimation (MLE).

Let us denote with \( F_i \) the accumulated distribution function of the marginals; we already know that the probability density of our multivariate distribution is given by:

\[
f_u(x_1, \ldots, x_n) = \frac{\partial^n}{\partial x_1 \cdots \partial x_n} C_{u,R}^n (F_i(x_1), \ldots, F_n(x_n))
\]  

(2.10)

where \( C_{u,R}^n \) is the chosen copula and \( \nu \) is the degree of freedom.

The likelihood function is defined as:

\[
L(\nu) = \prod_{j=1}^{h} f_u(x_{i,j}, \ldots, x_{n,j})
\]  

(2.11)

where \( h \) is the number of data in our sample. The maximum likelihood method states that the value \( \nu = \nu_{\text{max}} \) at which the likelihood function reaches its maximum can be assumed as a good estimator of the parameter \( \nu \) we want to estimate.

We can alternatively maximize the so called log-likelihood function:

\[
l(\nu) = \log L(\nu) = \sum_{j=1}^{h} \log f_u(x_{i,j}, \ldots, x_{n,j})
\]  

(2.12)

which can be written:

\[
l(\nu) = \sum_{j=1}^{h} \log \left( \frac{\partial^n}{\partial u_1 \cdots \partial u_n} C_{u,R}(u_1, \ldots, u_n) \bigg|_{u_1 = F_i(u_{i,j}), \ldots, u_n = F_n(u_{n,j})} f_i(x_{i,j}) \cdots f_n(x_{n,j}) \right)
\]  

(2.13)
Some simplifications give:

\[
l(\nu) = \sum_{j=1}^{h} \log \left( \frac{\partial^n}{\partial u_1 \cdots \partial u_n} C_\nu(u_1, \ldots, u_n) \right)_{u_i = F_i(x_{n,j}) \ldots u_n = F_n(x_{n,j})} + \sum_{j=1}^{h} \left( \log f_i(x_{n,j}) + \cdots + \log f_n(x_{n,j}) \right)
\]

As the second sum in the last expression does not depend on the parameter \( \nu \), we only have to maximize the modified log-likelihood function:

\[
\hat{l}(\nu) = \sum_{j=1}^{h} \log \left( \frac{\partial^n}{\partial u_1 \cdots \partial u_n} C_\nu(u_1, \ldots, u_n) \right)_{u_i = F_i(x_{n,j}) \ldots u_n = F_n(x_{n,j})}
\]

with respect to the parameter \( \nu \).

Nevertheless, we can choose several degrees of freedom and compare then the different results.

Finally, we can generate random numbers from the Student t-copula through the following algorithm:

- find the Cholesky decomposition \( A \) of the correlation matrix \( R \);
- simulate \( n \) independent random variates \( z = (z_1, \ldots, z_n) \) from the standard normal distribution;
- simulate a random variate \( s \) from \( \chi_\nu^2 \) distribution, independent of \( z \);
- determine the vector \( y = A \times z \);
- set \( x = \frac{\sqrt{D}}{\sqrt{s}} y \);
- determine the components \( u_i = t_\nu(x_i) \quad i = 1, \ldots, n \). The resultant vector is \((u, \ldots, u)^T \circ C_{\nu,R}^n\).

3. Loss ratio modelling and risk measures

We build for each CoI the loss ratio in the following way:
where $IC_{i,t}$ represents incurred claims and $EC_{i,t}$ represents earned premium during the period $(t; t + 1)$ for the class of insurance $i$. We then define the aggregate loss ratio as $LR_i = \sum_{j=1}^{5} \xi_j \cdot LR_{i,j}$, where the weights $\xi_j$ are all equal to $0.20$; the choice of $\xi_j$ has the aim of highlighting the role of correlations in the economic capital valuation; their sum is equal to one; in this way they represent the composition quotas of the earned premiums of the selected company.

As already said, the calculation of the economic capital of the company is based on the aggregate loss ratio of the company; it’s obvious it should take into consideration correlations amongst the five CoIs.

At this purpose, we use a dependence structure given by copula functions. We compare results obtained with the Gaussian copula and the Student $\text{t}$-copula (with several degrees of freedom).

We perform then a Monte Carlo simulation with $N$ scenarios. The simulation steps are the following:

I. estimate the parameters of the marginal distributions (loss ratio distribution for each class of insurance);

II. estimate the correlation matrix;

III. generate $N$ random correlated loss ratios $\overline{LR}_{i,j}$ (with $i = 1, \ldots, 5$ and $j = 1, \ldots, N$) from the selected copula;

IV. determine for each scenario the aggregate loss ratio $\overline{LR}_j = \sum_{i=1}^{5} \xi_i \cdot \overline{LR}_{i,j}$;

V. determine the statistics of the $N$-vector we obtained. In particular we determine some risk measures such as VaR and CVaR.

Let us fix a confidence level $\alpha \in (0,1)$. The VaR and CVaR values for the loss random variable $X$ at probability level $\alpha$ with cdf $F_X(x)$ will be defined as:

$$VaR = \min \left\{ \varsigma \in \mathbb{R} : F_X(\varsigma) \geq \alpha \right\} \quad CVaR = E[X \mid X > VaR] \quad (3.2)$$

In the first definition, VaR turns out to be the left endpoint of the nonempty interval consisting of the values $\varsigma$ such that $F_X(\varsigma) = \alpha$.
Besides we deduce that the probability that \( X \geq \text{VaR} \) equals \( 1 - \alpha \). Consequently, CVaR is seen as the conditional expectation of the loss associated with \( X \) relative to that loss being equal to VaR or greater.

The discretized version of the conditional VaR is the following:

\[
CVaR = \text{VaR} + \frac{1}{N \cdot (1 - \alpha)} \sum_{j=1}^{N} \left[ X - \text{VaR} \right] \\
(3.3)
\]

The algorithm for the simulation in the independence case is similar to the previous one and may be obtained by eliminating the step II and by substituting the step III with “generate \( N \) random independent loss ratios \( LR_{ij} \) (with \( i = 1, \ldots, 5 \) and \( j = 1, \ldots, N \)).”

In the independence context, the risk measures (VaR and CVaR) should reach their lower bound; this is equivalent to consider the so called independence copula:

\[
C(u_1, \ldots, u_n) = \prod_{i=1}^{n} u_i \\
(3.4)
\]

The upper bound is reached when we consider perfect dependence amongst classes of insurance. In this last case, the aggregate VaR (or CVaR) is just the sum of VaR for each class of insurance (the random vectors \( LR_i \) are then called comonotonic):

\[
\text{VaR}_c = \sum_{j=1}^{N} \text{VaR}(LR_j) \\
(3.5)
\]

Performing the numerical application, we will show that risk measures which consider real correlations stay between a lower bound (independence case) and an upper bound (perfect correlation).

In this framework, a diversification benefit can be assessed by comparing the real correlation state with the perfect correlation state. We can estimate it as \( DB = \text{VaR}_c - \text{VaR} \). In our numerical example, diversification benefit is expressed in a percent form and then transformed into monetary form (by considering the aggregate earned premiums).

4. A Numerical Example
We collected data of incurred claims and earned premiums for five classes of insurance from Italian non life insurance companies for the period 1980 - 2004.

As already said in section 1, the five classes of insurance are:

1. Accident
2. Land vehicles
3. Motor vehicle liability
4. Fire and natural forces
5. Third party liability

Mixing these data in a proper way, we have built an “average” insurance company Z.

The next graph shows the historical series of the loss ratios of each CoI in the described period (1980 – 2004).

**Graph 1 - The historical series of loss ratios**

We determine for each class of insurance the best fitting distribution using standard tools (such as @Risk). We found the following distributions (some distributions have an additional shift parameter):

**Table 1 - The best fitting distributions for the loss ratios of the five CoIs**
The density distributions for each CoIs are the following:

**Graph 2 - pdf for each CoIs**

![Graph 2 - pdf for each CoIs](image)

The empirical correlation matrix of the CoIs may be estimated using the historical series of the LRs; we obtained the following correlation table:

**Table 2 - The empirical correlation matrix for the loss ratios of the five CoIs**

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100%</td>
<td>68%</td>
<td>81%</td>
<td>78%</td>
<td>36%</td>
</tr>
<tr>
<td>2</td>
<td>68%</td>
<td>100%</td>
<td>54%</td>
<td>77%</td>
<td>-30%</td>
</tr>
<tr>
<td>3</td>
<td>81%</td>
<td>54%</td>
<td>100%</td>
<td>51%</td>
<td>50%</td>
</tr>
<tr>
<td>4</td>
<td>78%</td>
<td>77%</td>
<td>51%</td>
<td>100%</td>
<td>-6%</td>
</tr>
<tr>
<td>5</td>
<td>36%</td>
<td>-30%</td>
<td>50%</td>
<td>-6%</td>
<td>100%</td>
</tr>
</tbody>
</table>
Nevertheless, empirical correlations are not reliable in general, due to the few data we have at our disposal.
Consequently, we decided to compare empirical correlations with the “logical” values of the coefficients we previously described (section 1) to eliminate some false correlations. This comparison deals to the following matrix:

Table 3 - The coherent correlation matrix for the loss ratios of the five CoI

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100%</td>
<td>0%</td>
<td>75%</td>
<td>20%</td>
<td>0%</td>
</tr>
<tr>
<td>2</td>
<td>0%</td>
<td>100%</td>
<td>45%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>3</td>
<td>75%</td>
<td>45%</td>
<td>100%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>4</td>
<td>20%</td>
<td>0%</td>
<td>0%</td>
<td>100%</td>
<td>0%</td>
</tr>
<tr>
<td>5</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>100%</td>
</tr>
</tbody>
</table>

The degrees of freedom \( \nu \) (that individuate the best Student Copula) are calculated with a log-likelihood estimation that gives \( \nu = 5 \). The following graph represents graphically the log-likelihood vs. degrees of freedom:

Graph 3 - The estimation of degrees of freedom of student copula

To quantify the capital requirement and the diversification benefit we perform a Monte Carlo simulation with 50,000 scenarios. We consider the Gaussian copula and the Student copula with 10, 5 and 1 degrees of freedom respectively. The degree of freedom of the Student copula influences the tail dependence of the distribution. A low degree of freedom leads to high tail dependence while if we consider \( \nu \to \infty \), the Student copula tends to the
Gaussian copula, which has no tail dependence. The student copula with one degree of freedom is also called the Cauchy copula. Results coming from the adoption of the different copula dependence structures are then compared with results coming from the independence and the comonotonic assumptions.

As already said, we assume that the different classes of insurance are equally weighted, in order to estimate efficiently correlation effects. The correlations used are those showed in the previous table 3.

The following table shows the results obtained performing the Monte Carlo simulation.

**Table 4 - Statistics and risk measures of the company using several copula functions**

<table>
<thead>
<tr>
<th></th>
<th>Indep.</th>
<th>Gauss</th>
<th>Student t10</th>
<th>Student t5</th>
<th>Cauchy</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>mean</td>
<td>0.7532</td>
<td>0.7534</td>
<td>0.7528</td>
<td>0.7527</td>
<td>0.7538</td>
<td></td>
</tr>
<tr>
<td>std.</td>
<td>0.0446</td>
<td>0.0521</td>
<td>0.0523</td>
<td>0.0525</td>
<td>0.0541</td>
<td></td>
</tr>
<tr>
<td>skewness</td>
<td>0.6029</td>
<td>0.5888</td>
<td>0.7555</td>
<td>0.8678</td>
<td>1.4683</td>
<td></td>
</tr>
<tr>
<td>kurtosis</td>
<td>3.7432</td>
<td>3.6195</td>
<td>4.5671</td>
<td>5.1113</td>
<td>8.5097</td>
<td></td>
</tr>
<tr>
<td>VaR 95%</td>
<td>0.8327</td>
<td>0.8464</td>
<td>0.8461</td>
<td>0.8462</td>
<td>0.8523</td>
<td>0.9298</td>
</tr>
<tr>
<td>CVaR 95%</td>
<td>0.8606</td>
<td>0.8781</td>
<td>0.8821</td>
<td>0.8864</td>
<td>0.9067</td>
<td>0.9981</td>
</tr>
<tr>
<td>VaR 99%</td>
<td>0.8786</td>
<td>0.8988</td>
<td>0.9033</td>
<td>0.9130</td>
<td>0.9396</td>
<td>1.0400</td>
</tr>
<tr>
<td>CVaR 99%</td>
<td>0.9047</td>
<td>0.9264</td>
<td>0.9423</td>
<td>0.9525</td>
<td>0.9958</td>
<td>1.1042</td>
</tr>
</tbody>
</table>

We may appreciate the diversification benefit by comparing the results of the various copulas with those deriving from the comonotonic assumptions.

**Table 5 - Relative differences of VaR and CVaR with the comonotonic assumption**

<table>
<thead>
<tr>
<th>Saving</th>
<th>Indep.</th>
<th>Gauss</th>
<th>Student t10</th>
<th>Student t5</th>
<th>Cauchy</th>
</tr>
</thead>
<tbody>
<tr>
<td>VaR 95%</td>
<td>12%</td>
<td>10%</td>
<td>10%</td>
<td>10%</td>
<td>12%</td>
</tr>
<tr>
<td>CVaR 95%</td>
<td>16%</td>
<td>14%</td>
<td>13%</td>
<td>13%</td>
<td>16%</td>
</tr>
<tr>
<td>VaR 99%</td>
<td>18%</td>
<td>16%</td>
<td>15%</td>
<td>14%</td>
<td>18%</td>
</tr>
<tr>
<td>CVaR 99%</td>
<td>22%</td>
<td>19%</td>
<td>17%</td>
<td>16%</td>
<td>22%</td>
</tr>
</tbody>
</table>

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The following table shows the results of a simplified sensitivity analysis performed by supposing that all the coefficients of the correlation matrix are equal to 0, 0.1, 0.2, ..., 1 (assuming a Student copula with $\nu = 5$).

As intuitive, both the risk measures are increasing functions.

### Table 6 - VaR and CVaR vs. correlation

<table>
<thead>
<tr>
<th></th>
<th>VaR 95%</th>
<th>CVaR 95%</th>
<th>VaR 99%</th>
<th>CVaR 99%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Indep</td>
<td>0.8327</td>
<td>0.8606</td>
<td>0.8786</td>
<td>0.9047</td>
</tr>
<tr>
<td>10%</td>
<td>0.8456</td>
<td>0.8886</td>
<td>0.9118</td>
<td>0.9599</td>
</tr>
<tr>
<td>20%</td>
<td>0.8562</td>
<td>0.9014</td>
<td>0.9285</td>
<td>0.9768</td>
</tr>
<tr>
<td>30%</td>
<td>0.8640</td>
<td>0.9144</td>
<td>0.9459</td>
<td>0.9937</td>
</tr>
<tr>
<td>40%</td>
<td>0.8765</td>
<td>0.9287</td>
<td>0.9591</td>
<td>1.0132</td>
</tr>
<tr>
<td>50%</td>
<td>0.8861</td>
<td>0.9417</td>
<td>0.9702</td>
<td>1.0300</td>
</tr>
<tr>
<td>60%</td>
<td>0.8961</td>
<td>0.9544</td>
<td>0.9872</td>
<td>1.0481</td>
</tr>
<tr>
<td>70%</td>
<td>0.9026</td>
<td>0.9644</td>
<td>1.0001</td>
<td>1.0580</td>
</tr>
<tr>
<td>80%</td>
<td>0.9130</td>
<td>0.9729</td>
<td>1.0070</td>
<td>1.0774</td>
</tr>
<tr>
<td>90%</td>
<td>0.9199</td>
<td>0.9850</td>
<td>1.0212</td>
<td>1.0861</td>
</tr>
<tr>
<td>Comonot.</td>
<td>0.9298</td>
<td>0.9981</td>
<td>1.0400</td>
<td>1.1042</td>
</tr>
</tbody>
</table>

Finally, we built the efficient frontier in the plane LR – CVaR.
It is obtained solving the following optimization problem:

\[
\begin{align*}
\text{Max } E \left( \sum_{i=1}^{s} \xi_i \cdot LR_i \right) \\
CVaR &= \tau \\
\sum_{i=1}^{s} \xi_i &= 1
\end{align*}
\]

This problem consists in maximizing the expected aggregate loss ratio (varying the weights $\xi_i$, which must sum up to one) with a fixed conditional VaR level $\tau$.

We then represent graphically the efficient frontier:
The efficient frontier may be used to manage the total risk of the company and to individuate the best composition of the insurance business amongst the various CoIs line in correspondence of a specified economic capital requirement (CVaR).

5. Conclusions

Insurance companies face the problem of estimating economic capital requirements for risk management and solvency purposes. In this paper, we show that an approach consisting in ignoring real correlations amongst the several classes of insurance can lead to overestimating consistently the economic capital. A dependence structure given by copula functions turns out to be the optimal tool for managing correlations amongst the classes of insurance of the company.

Besides, we take into account a coherent risk measure, the CVaR, for constructing an efficient frontier (see Rockafellar & Uryasev, 2002) that may be used to optimise the relation between expected loss ratio and economic capital (represented by CVaR).
REFERENCES


