MONEY DEMAND: THEORIES AND ESTIMATION METHODS.  
A FRACTIONAL COINTEGRATION APPLICATION

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1. Introduction

Money demand is an economic theme, which has fascinated economists over the centuries and no unique result has been ever reached. Money demand, and money allocation in portfolio depend on the definition of money and wealth and on the possible combinations, depending on technology available and risk attitude. This paper surveys theoretical and empirical approaches to the theme and addresses it using a different estimation technique from traditional papers (i.e. fractional cointegration). Futures represent the widest and biggest innovation of financial markets; modern monetary economics should include financial innovation in the money demand function since it contributes to provide stability. More in details, we are interested in the relationship among (real) money holdings, income, the interest rate (on Federal Funds), and Futures, which is not instantaneous.

2. Money Demand Theories Survey

Using very simple notation, we can synthesise the evolution of money demand specifications and start with the well know quantitative theory of money \((MV = PQ)\), moving to the Fisher interpretation of it \((MV(r) = PQ)\) and then look at the Keynesian liquidity preference \((M^d = (r,Y))\) where money holdings are not only function of income (or consumption), but depend also on the alternative investment possible, following the speculative motive to hold money, together with precautionary and transactional. Tobin introduced

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the concept of average money holdings \( M = (2bT/r)^{1/2} \) where \( b \) is the brokerage charge to convert bonds into money, \( r \) is the interest rate and \( T \) is the number of transactions (i.e. the square-root law).

Financial innovation and evolving regulation might change the process of allocation of financial wealth in the form of money or other assets, and empirical and theoretical studies follow this process and try to find stable and meaningful functions over time and countries\(^1\).

Money demand and its relationship with growth and inflation are central themes in modern monetary and financial policy, but a stable function is the basic tool to identify the correct relationship with these final goals.

Following Barro and Santomero (1972) and Coenen and Vega (1999) we observe that a stable representation of the money demand should include alternative assets’ return to explain portfolio shifts and allocations in the short run. Barro and Santomero introduced, much before Regulation Q considered them, interest bearing deposits as assets explaining the behaviour of money demand of households in the US. Coenen and Vega more recently explain how the European money demand can be considered stable after introducing some financial assets, weighted with their returns and risk; these financial asset are representative of the European financial markets.

### 3. Money Demand and Financial Innovation

The Keynesian money demand \( M^d = (r,Y) \) is enriched with innovation \( (r^*) \) so that it can be represented implicitly as \( M^d = (r,Y,r^*) \). Financial innovation is an opportunity for investors, who, on the market, decide which innovation is going to survive and evolve, (i.e. vote with their feet) and monetary authority, which looks at the demand for money and its stability as an explanatory variable together with consumption and investments, should consider these portfolio shifts and their (adverse) effects.

Central banks are independent from the national Government and Treasury because of their ability to control targets, but financial innovation modifies this ability in the short run. Some monetary authorities have chosen particular (weighted) indexes to look at, like the Monetary Condition Index (MCI), given the impossibility to use money aggregates as instruments\(^2\). The only instrument for monetary policy is the interest rate, given flexible exchange rates regimes worldwide.

The Bank of Italy (1995a) states that, according to the money view, perfect substitutability between credit and bonds is the base for money manoeu-

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2 MCIs are neither instruments nor targets; they represent the weighted goals of monetary policy, in terms of interest and exchange rates.
vre to be effective (using interest rates). Derivatives increase markets’ liquidity and substitutability, and increase the speed of the transmission mechanism of monetary impulses; the distributive effect has to be considered. In fact, if individual risk can be shifted, at a macro level it cannot be cancelled, and it is of central importance to understand if those bearing more risks are really able to manage them (i.e. ordered market conditions). According to the credit view, imperfect substitutability between credit and bonds is supposed and the introduction of derivatives, highly substitutable with bonds and credit, can alter dramatically monetary policy actions and effects. The Bank concludes that a credit-based monetary policy is not going to be effective facing financial innovation, while interest rate manoeuvres aiming at ordered market conditions, taking into consideration the evolution of financial markets, and quasi-money instruments, are probably the best development of modern central banking.

Our aim is not to sustain that central banks should act on derivatives markets to stabilise the money demand since we believe in a radical change in money government and management, which has already taken place in many economic areas of the world; policy goals should be ordered market conditions, good monitoring and supervision systems, financial stability and transparency, and not simple quantitative rules or direct interventions on markets. Since policy variables are linked with intermediate targets (money, consumptions, asset prices) in a dynamic and two directions way, the final effect of quantitative operations could be different from the expected. An example is the policy developed by the Central Bank of New Zealand after the Asian crisis, which was theoretically correct but in practise contributed to recession and the spread of the crisis in the area. Quantitative goals have, in our view, to be replaced with qualitative rules, and accountability is an example. The only speaking of Alan Greenspan, Chairman of the Federal Reserve Bank, about rising interest rates in 2004 and early 2005 moves stock markets and credit as if he already did. This is the environment we face and try to deal with.

The use of derivatives for monetary purposes (futures, forwards and options) is also a cost-lowering way of managing interest and exchange rates (Cf. Bank of Italy, cit., pp. 84-85), and is not prohibited by current national and international regulation.

Pawley (1992) observes that financial innovation needs to be modelled into the demand for money as to identify a stable function. Broad demand for money is influenced by the change in the own rate of money; this elasticity is of central importance for money control and has not been sufficiently considered. Elasticity of the rate and substitutability represent therefore the core problem of modern monetary economics in the globalised economy.

Savona (2002) links the liquidity paradox stated by Keynes (1936), “everyone feels to be liquid but the market itself is not”, with the disruptive im-
pact of derivatives; central banks in the last 15 years have fought against excessive speculation\(^3\) using their function of lenders of last resort, and responding on market movements to avoid credit crunch (or even collapse). This is definitely a new way of managing monetary order, at home or internationally, and uses up-to-date instruments, like futures and options.

4. Time Series Investigations of the Money Demand

The money demand function, defined according to the theories as surveyed in the previous paragraphs, should be empirically implemented according to the country specific institutional framework, to the historical period, and to consider the effect of breaks (e.g. divorce between monetary and fiscal policy, war or currency union). The preferred empirical investigation procedure refers to time series, since across countries (i.e. cross section) the definition of main variables is not homogenous, leading to the complete lack of data and the impossibility of any reliable analysis.

Panel data estimates are un-developed in this field, since money demand basically refers to non-stationary variables, and techniques and theory are not yet able to deal with them.

Time series analysis can be started, after the check for the presence of unit roots\(^4\). Macroeconomic variables are often un-stationary, and the demand function should be expressed using the same root order; i.e. if all variables are I(1) a function could be expressed in terms of the levels; if one variable is I(2), we should take its first difference, which is I(1), to estimate its parameter with other I(1) variables\(^5\).

Simple money demand estimates on levels with the OLS provide unstable results and super-consistent coefficients. A simple log-linear representation of the money demand can be:

\[
(m_t - p_t) = \alpha + \beta y_t + \gamma r_t + \varepsilon_t.
\]  

(4.1)

Income or GDP \((y)\), the price index \((p)\), the interest rate \((r)\) and the money supply \((m\), narrow aggregates\) over the XX century and across industrialised countries (G7 for example) shown to be non-stationary, and often I(1);  

\(^3\) Some examples are the Federal Reserve and the LTCM in 1998, the Bank of Thailand and the Thai bath attack in 1997-98, the Bank of Italy and the lira in 1992, but many others could be named.  

\(^4\) Unit root tests are implemented in all econometric softwares, and are the Phillips-Perron or the Augmented Dickey Fuller. For further details on time series analysis of money demand refer to W. H. Greene (2003).  

\(^5\) Various authors suggest that using first difference of variables solves the econometric problem, but let to the loss of much information.
money demand estimates, being over long or short periods, have improved fast after the Engle and Granger procedure evolved. Friedman and Schwartz (1963) were the first to observe the existence of a strong correlation between money supply and the business cycle, Tobin added that this causal relationship could be reversed, and the Granger Causality test, introduced in the field by Sims (1972), finally cleared the way. Barro, with many co-authors, improved the analysis over the ’70s, by discerning the influence of real variables, shocks and un-anticipated components.

Modern money demand estimates can be split into short term analysis, which use the error correction approach (ECM), i.e. the Maximum Likelihood-ARCH estimator, and long term analysis, which use the Vector Auto Regression (VAR) or the Vector Error Correction Mechanism (VECM).

The ECM representation is based on relevant lags of variables, chosen according to their informative power with respect to the function (i.e. using the Akaike or the Schwarz-Bayesian information criteria). We need to start from a long-run relationship which provides the error correction term, and this is:

\[
(m-p)_t - y_t = \mu + \gamma i_t + \varepsilon_t,
\]

where income has a long run parameter equal to one, and the interest rate is exogenous; the left hand side is the inverse of the velocity of money (Lucas, 1988). A shown by Barro in various empirical exercises, considering separately long term and short term interest rates improves the estimate.

The short term ECM representation of money demand can be:

\[
\Delta(m-p)_t = \sum_{i=1}^{4} c_i \Delta(m-p)_{t-i} + \sum_{i=0}^{4} d_{1i} \Delta p_{t-i} + \sum_{i=0}^{4} d_{2i} \Delta y_{t-i} + \\
\sum_{i=0}^{4} d_{3i} \Delta RS_{t-i} + \sum_{i=0}^{4} d_{4i} \Delta RL_{t-i} + \lambda (m-p-y)_{t-i} + \gamma_1 RS_{t-i} + \\
\gamma_2 RL_{t-i} + d_i \phi + \omega_t,
\]

where \(RS\) is short term interest rate, \(RL\) is the rate on long maturities, \(d\) is a set of additional variables (constant and dummy variables to consider breaks), and \(\omega\) is the error term; the error correction is incorporated in \((m-p-y)\), which comes from the long term relationship. Since some variables are I(2) they have been taken at difference (i.e. \(m\) and \(p\)). The ARCH component considers the heteroskedasticity, and corrects the fact that the OLS estimator provides super-consistent results and is inefficient. The inefficiency is due to the covariance matrix of residuals, which does not vanish over time. The

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6 This example is taken from W. H. Greene (2003), page 660.
ARCH model, which uses the Maximum Likelihood estimator, corrects this problem, and provides efficient results.

The long run VAR representation following Lucas is instead:

\[(m - p - y)^* = \delta_0^* + \delta_2^* RS + \delta_3^* RL + \delta_4^* \Delta p. \quad (4.4)\]

VAR and VECM need a long length of time series data, which is not always available, especially for non-industrialised countries; the use of the GDP series in particular poses another limit, since it is available only at quarterly frequency.

The critics moved to VAR and VECM, which is a vector representation of the ECM, is that it looks for relationship between variables, after having set out which are exogenous, without any \textit{a priori} economic theory. Starting from \(n\) variables, we can end up with \(n-1\) vector representations, so that we have no further econometric mean to choose the best among them. The VAR procedure provides consistent representations, leaving the choice to the author, supported by the theory for example.

5. Long Memory Process and Fractional Cointegration

Generally, stochastic processes are divided into stationary and non-stationary. In this work we want to allow for the possibility, suggested by recent theories about stochastic processes, of nuances in stationarity and non-stationarity definition, opening the analysis to the intermediate cases represented by long memory processes. Consequently, we adopt a wide definition of cointegration, extending time series analysis to the frequency domain.

The process \(x_t\) is stationary, if it has zero mean, finite variance and autocovariances that do not depend on time, but only on the respective lags. Stationary processes can be generalized by ARMA (AutoRegressive Moving Average) models, such that:

\[x_t = \frac{\theta(L)}{\varphi(L)} e_t, \text{ with } \theta(L) = 1 + \theta_1 L + \theta_2 L^2 + ... + \theta_p L^p \text{ and } \varphi(L) = 1 - \varphi_1 L - \varphi_2 L^2 - ... - \varphi_p L^p \quad (5.1)\]

where \(\theta(L)\) and \(\varphi(L)\) are polynomials in the lag operator \(L\), \(e_t\) is white noise distributed (\(E e_t = 0\), \(E e_t^2 = \sigma^2\), and \(E e_t e_{t-\tau} = 0\) for \(\tau \neq 0\)), and \(\varphi(L)\) is invertible.
The process $x_t$ can also be explicitly written as a weighted sum of current and past innovations:

$$x_t = \sum_{j=0}^{\infty} \psi_j \varepsilon_{t-j} \quad (5.2)$$

where the coefficients $\psi$ represent how much of something happened in the past is currently remembered.

Stationary processes quickly “forget”; for them it holds that:

$$|\psi_j| \leq \text{constant} \cdot \psi^j, \text{ with } |\psi| < 1. \quad (5.3)$$

A general representation of a non-stationary process is the ARIMA’s (AutoRegressive Integrated Moving Average):

$$x_t = (1 - L)^{-d} \frac{\theta(L)}{\phi(L)} \varepsilon_t \quad (5.4)$$

Here, the variance of $x_t$ grows over time. It is worth noting that this process becomes an ARMA, once differentiated.

Suppose to have now:

$$x_t = (1 - L)^{-d} \frac{\theta(L)}{\phi(L)} \varepsilon_t \quad (5.5)$$

where $d \in \mathbb{R}$ and the fractional differencing operator is defined by:

$$(1 - L)^d = \sum_{k=0}^{\infty} \frac{\Gamma(k-d)L^k}{\Gamma(-d)\Gamma(k+1)} \quad (5.6)$$

Such a representation reduces to ARMA for $d = 0$ and to ARIMA for $d = 1$. For $d$ assuming non integer values, there is the so-called ARFIMA (AutoRegressive Fractionally Integrated Moving Average) representation; ARFIMA processes are stationary for $-\frac{1}{2} < d < \frac{1}{2}$ and mean-reverting for
\(-1 < d < 1\). Then, for \(\frac{1}{2} < |d| < 1\), they are non-stationary but mean-reverting\(^7\).

In particular, for \(d < 1\), the process is stationary, but it very slowly “forgets” and, for that reason, it is called long memory process.

In the case of ARFIMA models, the coefficients of the expansion can be proved to be:

\[
\psi_j \sim \text{constant} \cdot j^{d-1}
\]  

(5.7)

In conventional analysis, ARFIMA models are not considered; in effect, unit-root tests are designed to distinguish only the case of \(d\) taking the value of 1 or 0. In effect, the Dickey-Fuller and Phillips-Perron unit root tests only distinguish between stationarity and non-stationarity. They do not allow a proper assessment of what the importance of the non-stationary component of a time series can be. As they only consider two mutually exclusive alternatives, when the series of interest simultaneously has the feature of a random walk as well as those of stationary series, it is a proved and accepted result that these tests tend to accept the null hypothesis of non-stationarity.

Including intermediate cases can give more robustness to the analysis.

Why are we interested in long memory processes? Suppose we want to estimate the relationship:

\[
y_t = \beta \cdot x_t + u_t,
\]

(5.8)

where \(y_t\) and \(x_t\) are integrated of order \(d\) long memory processes

\[
\begin{pmatrix} y_t \\
 x_t \end{pmatrix} \sim I(d)
\]

(5.9)

\(y_t\) and \(x_t\) are fractionally cointegrated\(^8\), if it is possible to find a linear combination of them that lowers the memory of the process, such that:

\[
y_t - \hat{\beta} \cdot x_t = u_t \sim I(d'), \text{ with } d' < d,
\]

(5.10)

where \(\hat{\beta}\) is a certain estimate of the parameter \(\beta\).

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\(^7\) It is worth noting that each stationary process is mean reverting, but a mean-reverting process is not necessarily stationary.

\(^8\) See, for example, Robinson and Marinucci (1998).
This is the definition of cointegration, i.e. of long run equilibrium among integrated processes, that we are going to apply. In order to check whether this concept of cointegration applies to the variables involved in our analysis on money demand, we need to introduce time series analysis in the frequency domain.

In the frequency domain or Fourier space, a stochastic process is regarded as the sum of deterministic cycles with uncorrelated stochastic weights. The idea of spectral analysis of time series is to transform stochastic processes in order to abstract from their covariance function as much information as possible. In this view, the spectral density of a stationary process in the time domain is the frequency domain counterpart of a covariance function, that is its Fourier transform

\[
f(\lambda) = \frac{1}{2\pi} \sum_{\tau=-\infty}^{\infty} \gamma(\tau) e^{-i\lambda \tau}, \tag{5.11}\]

where \(\lambda\) is the frequency, \(i = \sqrt{-1}\), \(e^{i\lambda \tau} = \cos \lambda \tau - i \sin \lambda \tau\), and

\[
\gamma(\tau) = \int_{-\pi}^{\pi} f(\lambda) e^{-i\lambda \tau} d\lambda \tag{5.12}
\]

It is worth noting that: \(f(\lambda)\) has a period of \(2\pi\), that is \(f(\lambda) = f(\lambda + 2\pi j)\), where \(j\) is an integer; under certain conditions\(^9\), \(f(\lambda)\) assumes real values; \(f(\lambda) = f(-\lambda)\), such that the analysis of the spectral density can be restricted to the range \(\lambda \in [0, \pi]\).

\(f(\lambda)\) is a function of \(\lambda\) and expresses the value of the spectral density for each \(\lambda \in [0, \pi]\): the bigger is the spectral density at that frequency the bigger is the variance, then the process tends to repeat with that frequency.

For a white noise process \(z_t\), with \(\gamma_z(0) = \sigma_z^2\) and \(\gamma_z(\tau) = 0\) for \(\tau \neq 0\), \(f_z(\lambda) = \sigma_z^2 / 2\pi\), then its spectrum is flat and independent of \(\lambda\), because each frequency has an equal weight in determining the variance of the white noise.

For the ARMA process in (3), the spectral density is easily showed to be:

\[
f_x(\lambda) = \frac{\left|\theta(e^{-ik})\right|^2}{\left|\phi(e^{-ik})\right|^2} f_z(\lambda) = \frac{\left|\theta(e^{-ik})\right|^2}{\left|\phi(e^{-ik})\right|^2} \cdot \frac{\sigma_z^2}{2\pi}. \tag{5.13}
\]

Its spectrum is still bounded but not flat anymore.

A non-stationary process has an unbounded spectrum as frequencies go to zero.

Finally, the spectral density of a long memory process is bounded, but concentrated on the lowest frequencies, because it is dominated by long memory components.

6. Future into the Money Demand Function

The dynamics of short-run broad money demand adjusts to financial innovation, while the theory tells us that in the long-run money should be a stable function of income and interest rate\textsuperscript{10}.

Money demand should be modelled through the use of weighted monetary indexes, i.e. Divisia Index, introduced in the literature by Barnett (1992 and 1997) and developed and measured by the Federal Reserve Bank of St. Louis (they call it Monetary Services Index, MSI), because they address directly the problem of un-perfect substitutability contrary to traditional money aggregates, which are simple sums of assets.

The money demand function in the implicit form can be written as $(m/p) = f(r, y, \text{future})$, where $(m/p)$ is real cash balance (money demand), and is a function of interest rate ($r$), income ($y$), and the financial innovation ($\text{future}$) representative of market and portfolios in terms of liquidity, and open interest. The critic moved to the introduction of the future into the money demand function is that it is positively correlated with the underlying asset (the Dow Jones Industrial Average index) so that we could have confusion in the function, i.e. the coefficient we see is related with the underlying and not with the future, i.e. the futures is not an independent asset. This critic is valid iff there exist a risk-free rate, $r$, in the economy or if interest rates are zero. In a risky economy, derivatives are by definition independent of their underlying asset and enjoy specific pricing rules. The property of futures’ prices being correlated with the underlying is an efficiency characteristic and is called price discovery effect\textsuperscript{11}, but should not be confused with the independency; main differences, however, are the dimensions of the two markets, the liquidity degree, risk profile and costs-profits. We know that the derivatives’ market is hundreds of times bigger than the underlying. Liquidity of futures markets is the same or higher that the underling markets’; in some cases the underlying asset does not exist as such (like in the DJIA case) and then futures have higher liquidity by definition. Looking at costs and potential profit, the leverage effect of futures allow for higher (potential) profit and lower costs of in-

\textsuperscript{10} See Keynes (1936), chapp. 13 and 15, and Walsh (1998).

\textsuperscript{11} See J. Hull 2002 for math details.
vestments. Risk profile of futures is the same as the underlying, given matching price and price discovery effect, but because of their high liquidity and general efficiency, they are preferred for hedging activity. Moreover, futures and traditional stock exchange show different economic functions (derivatives exhibit leverage, hedging and substitutability)\textsuperscript{12}, while hedging and leverage cannot be exploited in the same way in the underlying market. We can conclude that they represent different assets and satisfy different functions in the money demand and asset allocation of investors.

Given that we want to enrich the specification of money demand function and include an innovation, which represents market evolution, we should include a liquid future, representative of the US economy, not highly volatile, used for hedging or speculation, and showing normal costs (in terms of bid-ask spread) on the market. The future will be included in the estimation over the period it contributes to lower financial volatility (1999 to 2000) so that we can detect its stabilising ability.

7. An Empirical Example on the US

Our aim is to check if a long-run equilibrium among money, income and prices exists and to understand if in a period of high instability futures play a crucial role in re-addressing these variables towards their equilibrium path.

Thus, the model to be estimated is:

\[
M_t / P_t = \alpha + \beta \cdot Y_t + \gamma r_t + \delta \cdot \text{Futures}_t + \delta' \cdot \mathbb{1}[t \in (a, b)] \cdot \text{Futures}_t + u_t \tag{6.1}
\]

where we use the monetary services index (MSIM3)\textsuperscript{13} for \( M_t \), the consumers price index (P)\textsuperscript{14} for \( P_t \), the industrial production index (INDUPROD)\textsuperscript{15} for \( Y_t \), and the Federal Fund rate\textsuperscript{16} (FEDFUND) for \( r_t \), the settlement price of future contract continuous rolled over on the Dow Jones Industrial Average Index (FUTURE)\textsuperscript{17} for \( \text{Future}_t \), and \( u_t \) is an error term. \( \mathbb{1}[t \in (a, b)] \) (DUM-FUT) is an indicator function that introduces a structural break in the model for the period of high instability denoted by the time interval \((a, b)\), which

\textsuperscript{12} For a wider discussion of economic functions of derivatives, see Savona (2002) and Vrolijk (1997), for monetary effects.

\textsuperscript{13} Source: Federal Reserve Bank of Saint Louis.

\textsuperscript{14} Source: Datastream.

\textsuperscript{15} Source: Datastream.

\textsuperscript{16} Source: Federal Reserve Bank of Saint Louis.

\textsuperscript{17} Source: Datastream.
started on 1/1999 up to 12/2000 when it takes on value 1 (0 otherwise). It has the meaning of a slope-correcting dummy, a significant value which underlines the powerful role of futures in determining the long-run equilibrium among the variables of interest. We added a dummy variable (DUMTT) to take into account the Twin Towers effect on financial and money markets in the US.

All data are monthly and start in October 1997 up to December 2004, because the future contract started on that date.

According to our theory, we expect that \( \beta \) is positive and substantially different from 0, that \( \delta \) is not substantially different from 0, and that \( \delta' \) assumes a positive and significant value, because we want to show that futures play a role in re-establishing the equilibrium in high instability periods.

In Figg. 1-4, the spectral density of each variable is reported. Each spectrum shows to have a high bounded peak at the lowest frequencies, denoting its long memory nature.

In choosing the coefficient estimation technique, we had to challenge two problems: variables simultaneity and the short sample period available. In effect, the relationship among these variables is not instantaneous; for example, disturbances in Federal Fund rate do not instantaneously transmit to money, or income.

Different ways to identify and test for a long-run relationship among variables are suggested by cointegration techniques. Lots of single equations and multiple equations models are available, each of them with its own advantages and drawbacks:

1) the Engle-Granger’s two-step procedure is based on a mechanism that do not take into account the problems of variables’ endogeneity and it also imposes untested common factor restrictions, that dramatically lower the power of tests, in the event of any of them being false;
2) VAR system, like in Johansen’s full-information maximum likelihood cointegration, allows to address the issue of simultaneity. However, it, being based on a maximum-likelihood estimation technique, extends specification problem linked to any single equation to the whole system;
3) Instrumental Variables techniques allow to address the issue of simultaneity of variables;
4) OLS estimators are super-consistent with non-stationary variables (converge to the real parameters values at a faster rate, \( n \) instead of \( \sqrt{n} \)).

Spectral densities refer to detrended series, since spectral analysis does not distinguish between deterministic and stochastic trends.
but its asymptotic distribution is severely biased, such that it makes it impossible any statistical inference;

5) Phillips-Perron Fully Modified OLS (FM-OLS) method that uses the super-consistency property of OLS estimators and accurately calculates the asymptotic distribution of the estimator of a cointegration relationship, allowing for statistical inference \(^{19}\) but with nonstationary time series requires particular attention to the choice of instruments \(^{20}\).

Each of these methods looks for a cointegration relationship among non-stationary variables. What is common to all of them is just the definition of stationarity and non-stationary process.

Conventional analysis only distinguishes between stationary and non-stationary processes.

Following Phillips and Hansen’s Monte Carlo results (1990) showing a better small sample properties of FM-OLS estimators with respect to the others, our choice fall on it. FM-OLS, synthetically, consists of correcting the asymptotic distribution of the OLS parameter estimator for the bias terms due to the dependence among variables \(^{21}\).

**Tab. 1 - Fully Modified Phillips-Hansen Estimates**

<table>
<thead>
<tr>
<th>Regressor</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>T-Ratio[Prob]</th>
</tr>
</thead>
<tbody>
<tr>
<td>INTERCEPT</td>
<td>-.5175</td>
<td>.15411</td>
<td>-3.3579 [.000]</td>
</tr>
<tr>
<td>INDUPROD</td>
<td>.0066332</td>
<td>.3020e-3</td>
<td>21.9572 [.000]</td>
</tr>
<tr>
<td>FEDFUND</td>
<td>-.071765</td>
<td>.020848</td>
<td>-3.4423 [.000]</td>
</tr>
<tr>
<td>FUTURE</td>
<td>.4168e-5</td>
<td>.8889e-6</td>
<td>.46895 [.641]</td>
</tr>
<tr>
<td>DUFUT</td>
<td>.3717e-4</td>
<td>.9806e-5</td>
<td>3.7902 [.000]</td>
</tr>
<tr>
<td>DUMTT</td>
<td>-.051548</td>
<td>.17980</td>
<td>-2.8670 [.006]</td>
</tr>
</tbody>
</table>

Truncation lags order chosen by running the regression for all possible lags and by choosing the order that gives the *whiter* regression error.

The FM-OLS estimate confirms all our expectations about parameters signs and statistical significance.

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\(^{19}\) See for example Hamilton (1994).

\(^{20}\) See for example Phillips and Hansen (1990).

\(^{21}\) *Ibidem.*
Actually, the spectral density of the regression residuals (Fig. 5) shows a peak at the lowest frequencies, but sharper than the ones showed by the original time series in Figg. 1-4. Hence, it shows to have less memory than the generating processes.

The last result, according to the definition of cointegration adopted, shows that a long-run relationship among the considered variables exists, and that it fulfils in the signs and the statistical significance of regression coefficients the hypotheses maintained. The magnitude of the Federal Fund is not directly comparable with that of future dummy, since the first is a rate and the second a price; signs are as expected and the substitution effect in real money balance is detected. Dummy variables are both significant, so that the Twin Towers effect influenced the money demand process over the period and corrects the intercept. The future dummy moreover, underlines the stabilising effect of futures over a period of high volatility of financial markets (1/1999-12/2000); without futures the money market subject to high volatility moves from its long-run equilibrium path and fails to come back. This is why the future coefficient is not statistically different from zero outside the instability period.

8. Concluding Remarks

Monetary theory aims at defining a stable representation of money demand function; this is very difficult, since financial markets introduce innovations on a daily basis and run faster than regulation, and theoretic analysis.

After a brief overview of econometric methods available in the field, we move to the empirical specification of money demand; country chosen for the investigation is the one where financial innovation is highly developed and available to all investors. Our aim is to look at an empirical specification of money demand, which includes one of the most traded, liquid financial asset, both at domestic and international levels (futures).

The Phillips and Hansen estimator fully exploits super-consistency of OLS estimator over small samples and periods, and differently from OLS, whose asymptotic distribution is not reliable for non-stationary variables, let estimate a regression and properly infer statistically. This is the main reason basing on which we choose this technique and not the others (e.g. cointegration), provided the existence of limits of data availability and length.

Results show that fractional cointegration is useful for underlying the role of future prices, i.e. financial innovation, in explaining instability in money demand. Futures help money demand function to come back to a stable long-run equilibrium path after instability periods. Traditional monetary literature has paid attention to modified money markets and institution, but a stable money demand function needs to be identified in order to provide meaningful
information about inflation pressures and financial order. The long-run equilibrium solution, i.e. money as a function of income and price, is confirmed by these results, but the inclusion of futures lets increase the descriptive power of the money demand function.

Future research should look at money demand by economic agents (households, public sectors, financial and non financial firms) using this innovative estimation technique, but a more comprehensive disclosure of data cannot be any more delayed by institutions looking after financial stability.
References


Appendix

Fig. 1: Real Money Balance

Fig. 2: Industrial Production

Fig. 3: Federal Fund Rate
Fig. 4: Future

Standardized Spectral Density Function of FUTURE Parzen window

\[ \text{Parzen} \]
\[ +2 \text{ S.E.} \]
\[ -2 \text{ S.E.} \]

Frequency

Fig. 5: Residuals

Standardized Spectral Density Function of FM-OLS regression residuals Parzen

\[ \text{Parzen} \]
\[ +2 \text{ S.E.} \]
\[ -2 \text{ S.E.} \]

Frequency

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